

Learning Team Activity: Compare/Contrast Models

Bathtub Model: $X' = 10 - 0.2X$	Logistic Growth Model: $X' = \overset{\text{green}}{b}X - \overset{\text{red}}{c}X^2$
Shark-Tuna model: $T' = -\beta ST + b_T T$ $S' = m\beta ST - d_S S$	

1. Find the outflow terms in each model. Mark these in red. How did you identify the outflow terms?
2. Find the inflow terms in each model. Mark these in blue. How did you identify the outflow terms?
3. Find the parameters in each model. Mark these in green. How did you identify the outflow terms?
4. Find the terms that show state variables interacting with one another. Mark these in yellow. How did you identify the outflow terms?

Class Activity: Eucalyptus & Koalas

5. Koalas (K) feed exclusively on eucalyptus (E). Use the following assumptions to write a differential equation model of the food chain.
- (1) Eucalyptus grows at a per-biomass (like per-capita) rate of 0.01.
 - (2) Due to shading, the per-biomass rate at which eucalyptus dies is proportional to the amount of eucalyptus with a proportionality constant of 0.02.
 - (3) Koalas consume eucalyptus at a per-capita rate proportional to the amount of eucalyptus available, with a proportionality constant of 0.05.
 - (4) The koala birth rate is proportional to the amount of eucalyptus they consume, with a proportionality constant of 0.15.
 - (5) Koalas die at a per-capita rate of 0.03



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For the situation above,

- Identify the state variable(s).
- Sketch a box-and-arrow diagram. Draw in all the connections, but if the connection is not a direct flow from one box to another with the same numbers, use a dotted line.
- Write a differential equation model

Learning Team Activity: Ants and Aphids

6. Ants and aphids have a well-documented relationship, as described below. Set up a differential equation model for the populations of ants (N) and aphids (P), based on the following assumptions:



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- (1) The aphids secrete a sugary substance called honeydew, which provides a food source for the ants. Therefore, the per-capita birth rate of the ants is proportional to the aphid population, with a proportionality constant of 0.03.
- (2) 2% of the ants die per day.
- (3) The aphids have a per-capita birth rate of 12% per day.
- (4) Due to crowding, the per-capita death rate of the aphids is proportional to the aphid population, with proportionality constant 0.001.
- (5) Aphids can also be killed by predators, but to protect their source of honeydew, the ants will aggressively defend the aphids: the more ants there are, the lower the risk of predation for the aphids. Thus, the per-capita predation rate for aphids is proportional to the inverse of the ant population, with a proportionality constant of 0.7.

For the mathematical model above,

- a. Identify the state variable(s)
- b. Sketch a box-and-arrow diagram. Draw in all the connections, but if the connection is not a direct inflow or outflow, use a dotted line. (Like we did with feedback diagrams.)
- c. Write a differential equation model