# Chapter 4 Review

**Theorem:** Let $S = \{a_1, \ldots, a_m\}$ be a set of vectors of $\mathbb{R}^n$. Let $A = [a_1 \ \ldots \ a_n]$ be a matrix and $T : \mathbb{R}^m \to \mathbb{R}^n$ be the linear transform defined by $T(x) = Ax$. Let $B$ be an echelon form of $A$. Then the following objects are equal:

- The set of vectors killed by $T$,
- $\{x : Ax = 0\}$ (this is the set of homogeneous solutions to $A$),
- null($A$),
- $\{x : T(x) = 0\}$,
- ker($T$),
- number of rows of all zeros in $B$,

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- The set of vectors hit by $T$,
- $\{T(x) : x \in \mathbb{R}^n\}$,
- range($T$),
- col($A$),
- span($S$),

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- dim(col($A$)),
- dim(range($T$)),
- dim(span($S$)),
- $m$ - nullity($A$) (rank-nullity theorem),
- $m$ - dim(ker($T$)),
- dim(row($A$)), (think of this as maximal number of linear independent equations in $Ax = 0$),
- number of pivots in $B$,

**Example:** Let $T(x) = Ax$, where $A$ is

$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ -2 & -4 & 1 & -3 \\ 1 & 2 & 2 & 4 \end{bmatrix}$$

and has reduce echelon form $B$ given by

$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- What is the range of $T$?
- What is the kernel of $T$?
- What is the row space of $A$?
- What is the rank of $A$?
- What is the nullity of $A$?
- Write the columns corresponding the free variables as a linear combination of the pivot columns.

- What is the general solution to $Ax = 0$?
- What is the general solution to $Ax = [2 - 3, 4]^t$?
- What is a vector not in the range of $T$?

**Example:** Answer all the same questions as above but for an invertible transform.

**Example:** Give an example of a linear transform $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1, 1, 0) = (1, 0)$ and $T(0, 1, 2) = (1, 2)$.

- What is the smallest possible rank such an example could be?
- What is the largest possible rank such an example could be?
- What is the smallest possible nullity such an example could be?
- What is the largest possible nullity such an example could be?