Basis and Dimension Round 2

Definition: A set $B = \{u_1, \ldots, u_m\}$ is a *basis* for a subspace S if

- B spans S,
- B is linearly independent

Theorem: Let $B = \{u_1, \ldots, u_m\}$ be a basis for a subspace S. Then every $s \in S$ can be written as a linear combination of $u_1, \ldots, u_m$ in a unique way.

Example: We'll use the same subspace as last time. Let $S \subseteq \mathbb{R}^4$ be the subspace spanned by $u_1 = (-1, 2, 3, 1), u_2 = (-6, 7, 5, 2), u_3 = (4, -3, 1, 0)$. From last class, we determined that a basis for S is given by $\{v_1, v_2\}$ where $v_1 = (-1, 2, 3, 1), v_2 = (0, 5, 13, 4)$. It is clear that $u_3 \in S$. How do we express u_3 as a linear combination of v_1, v_2? This amounts to solving $[v_1,\ v_2 | u_3]$.

```
v1 = vector([-1,2,3,1])
v2 = vector([0,5,13,4])
u3 = vector([4,-3,1,0])
A = matrix([v1,v2]).transpose()
A, u3
```

```
(
[-1  0]
[ 2  5]
[ 3 13]
[ 1  4], (4, -3, 1, 0)
)
```

```
A \ u3
```

```
(-4, 1)
```

Most sets of n vectors in $\mathbb{R}^n$ are a basis.

Example: Take S as before with basis $B = \{v_1, v_2\}$. How can we extend B to be a basis for $\mathbb{R}^n$?

- Eyeball it
- Add 2 random vectors

In this case, we see that $\{v_1, v_2, e_1, e_2\}$ will form a basis for $\mathbb{R}^4$. But so will $\{v_1, v_2, (-213, \pi, 4, 2), (4, \pi^2, 3, 4)\}$.