October 4

Announcements

- Section 1.1, 1.2 due tomorrow
- Worksheet 1 due Friday there are html issues, see pdf version
- Section 2.1, 2.2 due next Thursday
- Office hours today after class and tomorrow 12-1

Linear combinations and span

Definition: If u_1, u_2, \ldots, u_m are vectors and c_1, c_2, \ldots, c_m are scalars, then

$$c_1u_1 + c_2u_2 + \ldots + c_mu_m$$

is a linear combination of u_1, \ldots, u_m . Note that the constants can be negative or zero.

Definition: Let $S = \{u_1, u_2, \dots, u_m\}$ be a set of vectors. Then the span of S, span S, is the set of all linear combinations of $u_1, u_2, \dots, u_m\}$.

What vectors in \mathbb{R}^2 are a linear combination of (1,0) and (0,1)? In other words, what vectors are in the span of (1,0) and (0,1)?

What vectors in \mathbb{R}^2 are a linear combination of (1,2) and (0,1)? Talk about lines and averages here.

Is (3,4) a linear combination of (1,2) and (0,1)? In other words, is (3,4) in the span of (1,2) and (0,1)? In other words, does there exists $x_1, x_2 \in \mathbb{R}$ such that $x_1(1,2) + x_2(0,1) = (3,4)$? In other words, system of equations!

Every system of equation can be interpeted in this way.

Theorem: Let u_1, \ldots, u_m and v be vectors in \mathbb{R}^n . Then $v \in \text{span}(\{u_1, \ldots, u_m\})$ if and only if the linear system with augmented matrix $[u_1 \ u_2 \ \ldots \ u_m | v]$ has a solution.

The solution space can be expressed as a linear combination.

Theorem: Let u_1, u_2, \ldots, u_m be vectors in \mathbb{R}^n . If $u \in \text{span}(\{u_1, \ldots, u_m\})$, then $\text{span}(\{u_1, \ldots, u_m\}) = \text{span}(\{u_1, \ldots, u_m, u\})$.

When does a set of vectors span \mathbb{R}^n ?

Theorem: Let u_1, u_2, \ldots, u_m be vectors in \mathbb{R}^n . Let $A = [u_1 \ u_2 \ \ldots \ u_m]$ and $B \sim A$, where B is in echelon form. Then $\operatorname{span}(\{u_1, \ldots, u_m\}) = \mathbb{R}^n$ if and only if B has a pivot position in every row.

give outline of proof

We can write linear systems as Ax = b.