September 29

Announcements

- Section 1.2 due Thursday
- Fill out office hours preference form

Triangular Systems Round 2

Definition: A square system is triangular if leading variable of the ith equation is the ith variable.

Solutions to echelon systems

Recall that in a echelon system, if a variable is not a leading variable to any equation then it is called a free variable. Let's justify this with an example.

Consider linear system given by

$$x_1 + 2x_2 - x_3 + 3x_5 = 7 \tag{1}$$
$$x_2 - 4x_3 + x_5 = -2 \tag{2}$$
$$x_4 - 2x_5 = 1. \tag{3}$$

Here x_1, x_2, x_4 are leading variables and x_3, x_5 are free variables. Notice that I can add set x_3 and x_5 to any number and they will be a unique solution. For example, $x_3 = 3$ and $x_5 = 2$. This gives the triangular system

$$x_1 + 2x_2 - x_3 + 3x_5 = 7 \tag{4}$$
$$x_2 - 4x_3 + x_5 = -2 \tag{5}$$
$$x_3 = 3 \tag{6}$$
$$x_4 - 2x_5 = 1 \tag{7}$$
$$x_5 = 2, \tag{8}$$

which has a unique solution using back substitution. Let's solve this linear system in general. We can set $x_3 = s_1$ and $x_5 = s_2$. Here s_1, s_2 are parameters that determine our solution space.

$$x_1 + 2x_2 - x_3 + 3x_5 = 7 \quad (9)$$
$$x_2 - 4x_3 + x_5 = -2 \quad (10)$$
$$x_3 = s_1 \quad (11)$$
$$x_4 - 2x_5 = 1 \quad (12)$$
$$x_5 = s_2, \quad (13)$$

Work this out with class. Remark that the solution space is two dimensional.

With this technique, all echelon systems can be solved with back substitution if we ignore the constant equations like $0 = 0$ or $1 = 0$. We know how to handle the constant equations.

1.2 Linear Systems and Matrices

tldr: We can turn all linear systems into echelon systems without changing the solution space. We can organize the data of a linear system using matrices.

Augmented matrix

We can write a linear system as an augmented matrix. (give example in class).

Definitiion: The *leading term* of a row of a matrix is the leftmost nonzero term.

Definitiion: A matrix is in echelon form if * every leading term is in the column to the left of the leading term of the row below it * any zero rows are at the bottom

Elementary operations

We can perform a series of *elementary operations* to turn a general linear system into a echelon system without changing the solution space.

- Interchange the position of two equations. Swapping rows. (give example in class)
- multiply an equation by a nonzero constant. Multiplying a row by nonzero constant. (give example in class)
- add a multiple of one equation to another. add a multiple of a row to another. (give example in class)

The most important part about these operations is that they do not change the solution space. They do not change solution space. No changes to solution space. Solution space is the same. Same solution space.

Gaussian elimination

Definition: The *pivot positions* are positions that contain a leading term. The *pivot columns* are columns that contain a pivot position. A *pivot* is the value of a *pivot position.*

Algorithm: *Gaussian elimination* is performed as follows: * find the pivot position in the first row * use elementary row operators to eliminate all value under the pivot position * continue

work out example in class

Reduced echelon form

Definition: A matrix is in *reduced echelon form* if * it is in echelon form * all pivot positions contain a 1 * the only nonzero term in a pivot colum is in the pivot position

Algorithm: *Gauss-Jordan elimination* is performed as follows: * do Gaussian elimination * divide each row by the value of its pivot * eliminate all other values in pivot column.

work out example in class.

Homogenous linear systems

A linear system is homogenous if the numbers to the right of the equal sign are all zero. They always have the trivial solution