Worksheet 2

Due 10/13

1. We know how to obtain the general solution from a linear system. Let's try to reverse it. Find a linear system who's general solution is

$$(x_1, x_2, x_3, x_4) = (1, 2, 3, 4) + s_1(5, 6, 7, 8) + s_2(9, 0, 1, 2).$$

2. Suppose A is a matrix. Let v, w be distinct (meaning $x \neq y$) vectors that solve $Ax = 0$ so $Av = 0$ and $Aw = 0$ (0 here of course means the zero vector!). Let L be the line that passes through v and w. If u is on L, then $Au = 0$. Why? This exercise suggests that solution spaces are convex.

3. Let $z_1, z_2 \in \mathbb{R}$ and let $S = \{(1, z_1, z_2), (2, 1, 0), (1, 0, -1)\}$.
   - Find some values for z_1 and z_2 such that S spans $\mathbb{R}^3$.
   - Find some values for z_1 and z_2 such that S does not span $\mathbb{R}^3$.
   - Find all values for z_1 and z_2 such that S spans $\mathbb{R}^3$. (In the process of solving this problem, some of you will be tempted to divide by zero. Resist that temptation.)

4. Consider the following linear system that came from the book and the lecture.

$$2x_1 - 6x_2 - x_3 + 8x_4 = 0 \quad (1)$$
$$x_1 - 3x_2 - x_3 + 6x_4 = 0 \quad (2)$$
$$-x_1 + 3x_2 - x_3 + 2x_4 = 0. \quad (3)$$

   Using row reduction, we see that a general solution is of the form $x = s_1(3, 1, 0, 0) + s_2(-2, 0, 4, 1)$. Let $v_1 = (2, 1, -1), v_2 = (-6, -3, 3), v_3 = (-1, -1, 2), v_4 = (8, 6, 2)$.
   - Is $\{v_1, v_2, v_3, v_4\}$ is linearly independent set? The answer should be no.
   - Express v_1 as a linear combination of v_2, v_3, v_4.
   - Express v_2 as a linear combination of v_1, v_3, v_4.
   - Express v_3 as a linear combination of v_1, v_2, v_4.
   - Express v_4 as a linear combination of v_1, v_2, v_3.

5. Suppose $\{v_1, v_2, v_3\}$ is a linearly dependent set. Is it always the case that we can write v_1 as a linear combination of v_2 and v_3? If not, come up with a counterexample.

6. Come up with a inconsistent linear system whose associated homogenous linear system is consistent.