Worksheet 3

Due 10/20

1. During the October 13th lecture, I wrote down many statements equivalent to "S is a linearly independent set". Do the same for "S is a spanning set". The answer is in the notes but see what you can do from memory.
2. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation. We know that there exists a matrix A such that $T(x) = Ax$.
   - Suppose we know that $T(1, 0) = (2, 3, 4)$ and $T(0, 1) = (-1, 2, 1)$. Can we determine A? If so, what is it? If not, why not?
   - Suppose instead we know that $T(1, 0) = (2, 3, 4)$ and $T(2, 0) = (4, 6, 8)$. Can we determine A? If so, what is it? If not, why not?
   - Suppose instead we know that $T(1, 0) = (2, 3, 4)$ and $T(1, 1) = (-1, 2, 1)$. Can we determine A? If so, what is it? If not, why not?
   - Suppose instead we know that $T(x) = u$ and $T(y) = v$. Under what conditions on x and y, can we determine A?
3. Come up with a linear transform that is:
   - One-to-one and onto
   - One-to-one but not onto
   - Onto but not one-to-one
   - Not one-to-one nor onto
4. Is differentiation a linear transformation? The answer is yes. I just want you to think about why this is true.
5. Do a full exam from the exam archive here under test like conditions.