### Worksheet 3

# Due 10/20

1. During the October 13th lecture, I wrote down many statements equivalent to "S is a linearly independent set". Do the same for "S is a spanning set". The answer is in the notes but see what you can do from memory.

### ANSWER: See notes

- 2. Let  $T: \mathbb{R}^2 \to \mathbb{R}^3$  be a linear transformation. We know that there exists a matrix A such that T(x) = Ax.
  - Suppose we know that T(1,0) = (2,3,4) and T(0,1) = (-1,2,1). Can we determine A? If so, what is it? If not, why not?
  - Suppose instead we know that T(1,0) = (2,3,4) and T(2,0) = (4,6,8). Can we determine A? If so, what is it? If not, why not?
  - Suppose instead we know that T(1,0) = (2,3,4) and T(1,1) = (-1,2,1). Can we determine A? If so, what is it? If not, why not?
  - Suppose instead we know that T(x) = u and T(y) = v. Under what conditions on x and y, can we determine A?

#### ANSWER:

- Yes. The columns of the matrix are (2,3,4) and (-1,2,1).
- No. We don't know what T(0,1) is. There are infinitely many possibilities for A. Just set T(0,1) to be whatever you like.
- Yes. We need to determine what T(0,1) is. But (0,1) = (1,1) (1,0). So by linearity, T(0,1) = T(1,1) T(1,0) = (2,3,4) (-1,2,1) = (3,1,3).
- When x, y are spanning (which is equivalent to linearly independent here!). More on this later.
- 3. Come up with a linear transform that is:
  - One-to-one and onto
  - One-to-one but not onto
  - Onto but not one-to-one
  - Not one-to-one nor onto

## ANSWER:

- T(x,y) = (x,y).
- T(x) = (x, 0).
- T(x,y) = x.
- T(x,y) = (0,0).
- 4. Is differentiation a linear transformation? The answer is yes. I just want you to think about why this is true.
- 5. Do a full exam from the exam archive here under test like conditions.