Worksheet 5 - Due 11/3

1. Extend $\{(1,-1,0,0),(1,0,-1,0)\}$ to a basis for the subspace, W, defined by $w+x+y+z=0$. In other words, find a basis for W that includes $(1,-1,0,0)$ and $(1,0,-1,0)$.

2. Let P be the plane given by $2x+y+z=0$ in $\mathbb{R}^3$.
   (a) What is a normal vector to P?
   (b) Give a basis for $\mathbb{R}^3$ that includes a normal vector to P and 2 vectors that lie on P.
   (c) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transform that reflects all vectors across P. This means that $T(n) = -n$ whenver n is normal to P and $T(v) = v$ if v lies on P. Find A such that $T(x) = Ax$.
   (d) What is the rank of T? What is the nullity of T?

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transform defined by $T(1,1,1) = (1,0)$, $T(1,0,1) = (1,1)$, and $T(1,1,0) = (0,2)$.
   (a) Before doing a single computation, what can you already say about the rank and nullity of T?
   (b) Give a matrix A such that $T(x) = Ax$. You may express A as a product of matrices and their inverses.
   (c) What is the rank and nullity of T?

4. Give an example of each of the following. If it is not possible, write NOT POSSIBLE.
   (a) Find an invertible 3×3 matrix A and a 3×3 matrix B such that $rank(AB) \neq rank(BA)$.
   (b) Find two 3×3 matrices A and B, each with nullity 1 such that AB is the zero matrix.
   (c) Find two 3×3 matrices A and B, each with rank 1 such that AB is the zero matrix.
   (d) Find two 3×3 matrices A and B, each with nullity 2 such that AB is the zero matrix.
   (e) Find two 3×3 matrices A and B, each with rank 2 such that AB is the zero matrix.