Worksheet 8 - Never due

1. Give an example of each of the following. If it is not possible, write "NOT POSSIBLE".
   (a) Give an example of a basis of $\mathbb{R}^4$ such that each element lies in the hyperplane $2w+3x+y+z=0$.
   (b) Give an example of a basis of $\mathbb{R}^4$ such that each element lies in the hyperplane $2w+3x+y+z=1$.
   (c) Give an example of a matrix that is orthogonally diagonalizable but not diagonalizable.
   (d) Give an example of a matrix that is diagonalizable but not orthogonally diagonalizable.
   (e) Give an example of a nonzero matrix A such that $A^2=0$.
   (f) Give an example of a nonzero matrix A such that $A^2=I$.
   (g) Give an example of a nonzero matrix A such that $A^2=I$ and the nullity of A is 1.
   (h) Give an example of an orthogonal set that is not linearly independent.
   (i) Give an example of an orthogonal set that is not spanning.
   (j) Give an example of a 2×3 matrix whose rank is equal to its nullity.
   (k) Give an example of 2 matrices A and B such that $A^3=B^3$.
   (l) Give an example of 2 matrices A and B such that A and B each have nullity 1 but AB has nullity 0.
   (m) Give an example of 2 matrices A and B such that A and B each have nullity 0 but AB has nullity 1.
   (n) Give an example of a diagonalizable matrix that is not invertible.
   (o) Give an example of an invertible matrix that is not diagonalizable.
   (p) Give an example of a symmetric matrix that is not diagonalizable.
   (q) Give an example of a symmetric matrix that is not invertible.
   (r) Give an example of an orthogonal matrix that is not invertible.
   (s) Give an example of an invertible matrix that is not orthogonal.
   (t) Give an example of a matrix with distinct eigenvalues that is not invertible.
   (u) Give an example of a 3×3 orthogonal matrix with only one eigenvalue.
   (v) Give an example of a 3×3 matrix whose only eigenvalue is 2.
   (w) Give an example of a 3×3 invertible matrix whose only eigenvalue is 2.
   (x) Give an example of a matrix A and an eigenvalue λ such that the algebraic multiplicity of λ is less than the geometric multiplicity.
   (y) Give an example of a matrix A and an eigenvalue λ such that the geometric multiplicity of λ is less than the algebraic multiplicity.
   (z) Give an example of a matrix A and an eigenvalue λ such that the eigenspace is 0-dimensional.