NAME: _____

- *To get credit for a problem, you must show all of your reasoning and calculations.*
- *No cell phones or calculators may be used during the exam.*

Page	**Score**
1	/ 10
2	/ 10
3	/ 7
4	/ 12
5	/ 11
Total	/ 50

1. *(10 points)* A circle can be described by the equation $x^2 + y^2 = ax + by + c$. If the points $(1, 0), (-1, 2)$ and $(1, 4)$ are on the circle, what are a, b and c? (If you finish early, for +.001 bonus points find the center and radius of the circle.)

2. *(7 points)* Write the augmented matrix for the following system and put it in reduced echelon form:

$$\begin{aligned} 2x + 3y + z + w &= 0 \\ x + y + z + w &= -3 \\ z - w &= 5 \end{aligned}$$

3. *(3 points)* Give an example of a matrix in echelon form whose corresponding system of equations has no solutions.

4. *(3 points)* Compute $2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 7\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

5. *(4 points)* Consider the system of equations:

$$\begin{aligned} x + y + z &= 0 \\ 2x + y + 3z &= 2 \\ x - y - z &= 4 \end{aligned}$$

Find the matrix A and vector b, such that $Ax = b$ gives the same system of equations.

6. *(12 points)* Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 0 \\ 3 \\ 5 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 0 \\ 4 \\ -3 \\ 3 \end{bmatrix}$

(a) Is $\begin{bmatrix} 1 \\ 6 \\ -3 \\ 7 \end{bmatrix}$ in $\text{span}(v_1, v_2, v_3)$?

(b) Is $\text{span}(v_1, v_2, v_3) = \mathbb{R}^4$? Explain.

(c) Is $\{v_1, v_2, v_3\}$ linearly independent? Explain.

7. *(11 points)* Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $T(x) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 1 \\ 1 & 0 & 2 \end{bmatrix} x$.

   (a) Is T a linear transformation? Explain.

   (b) Is T one-to-one? Explain.

   (c) Is T onto, i.e., is the span of the columns of the matrix $\mathbb{R}^3$? Explain.