Math 308B **Midterm 1** **Spring 2017**

Your Name

Your Signature

Student ID #

Section (Tues.)	8:30	9:30	10:30
(circle one)	BA	BB	BC

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 5 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. (10 points) Solve the system of linear equations.

$$\begin{aligned} -2x_1 + 3x_2 + 7x_3 - 11x_4 &= -3 \\ x_1 \qquad - 2x_3 + \quad x_4 &= 3 \\ x_1 - \quad x_2 - 3x_3 + \ 4x_4 &= 2 \end{aligned}$$

Give two different particular solutions to the linear system.

2. (10 points) Find all values of x such that $\text{span}\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\} = \text{span}\{\mathbf{v_1}, \mathbf{v_2}\}$ where

$$\mathbf{v_1} = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{v_3} = \begin{bmatrix} 2 \\ x \\ -5 \end{bmatrix}.$$

3. (10 points) Determine whether the following set of vectors is linearly independent or linearly dependent. If the set is linearly dependent, express one of the vectors as a linear combination of the others.

$$\mathbf{u_1} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{u_2} = \begin{bmatrix} -2 \\ 3 \\ -3 \\ -3 \end{bmatrix}, \quad \mathbf{u_3} = \begin{bmatrix} -5 \\ 7 \\ -8 \\ -7 \end{bmatrix}, \quad \text{and} \quad \mathbf{u_4} = \begin{bmatrix} 3 \\ -4 \\ 6 \\ 4 \end{bmatrix}.$$

4. (10 points) Find a 3×5 matrix A, in *reduced* echelon form, with free variables x_3 and x_5, such that the general solution of the equation $A\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ is $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 4 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 0 \\ 5 \\ 1 \end{bmatrix}$,

   where s and t are real numbers.

5. (10 points) Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that

$$T\left(\begin{bmatrix} 3 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 1 \\ -7 \end{bmatrix}, \quad T\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

or explain why such a linear transformation cannot exist.