| Your Name | Your Signature | |
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| | | |
| Student ID # | | |
| | Section (Tues.) 8:30 9:30 10:30 | |
| | (circle one) BA BB BC | |

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 4 pages, plus this cover sheet. Please make sure that your exam is complete.

| Question | Points | Score |
|----------|--------|-------|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total | 50 | |

1. (10 points) Let
$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ -3 & 3 & 1 & 0 \\ 1 & 2 & -5 & -2 \end{bmatrix}$$
.

Calculate the inverse of B, or explain why it does not exist.

2. (10 total points) Determine whether each of the following is a subspace of \mathbb{R}^4 . Justify your answer.

(a) (5 points) The set
$$V$$
 of vectors of the form $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$, where $a+b+c=0$ and $b+c+d=0$.

(b) (5 points) The set W of vectors of the form $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$, where $a^2 + b^2 + c^2 + d^2 = 0$.

3. (10 points) Let
$$A = \begin{bmatrix} -3 & -6 & 6 \\ 3 & 6 & -3 \\ 0 & 0 & 3 \end{bmatrix}$$
.

Find the eigenvalues of A. Then find bases for the corresponding eigenspaces of the matrix.

4. (10 points) Find a linear transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$ with kernel $\{\mathbf{0}\}$, or explain why such a linear transformation cannot exist.

5. (10 points) Find matrices A and B such that rank(AB) > rank(A), or explain why such matrices cannot exist.