Parameterization by arclength round II

Problem: Parameterize some curve with respect to arclength from $t = a$ in the direction of increasing t.

1) Write

$$s(t) = \int_a^t |r'(t)|\, dt$$

2) Solve for t as a function of s.

3) The answer is

$r(s(t))$.

$$\int_a^t |\tfrac{d}{dt} r(s(t))|\, dt = \int_a^t |r'(s(t))| \cdot |s'(t)|\, dt = \int_a^t |r'(s(t))| \cdot |r(t)|\, dt$$

The normal and binormal vectors round II

Recall $T(t) = \frac{r'(t)}{|r'(t)|}$ so $|T(t)| = 1$. When a vector function s has the property $|s(t)| = c$, then $s(t) \cdot s'(t) = 0$. So $T(t) \neq 0$, then we can define the unit normal vector

$$N(t) = \frac{T'(t)}{|T'(t)|}.$$

The unit binormal vector is

$$B(t) = T(t) \times N(t).$$

Together they form the TNB frame. (coordinate system for vectors).

The plane determined by N and B at a point P on a curve C is called the normal plane of C at P. It consists of all lines passing through P orthogonal to T.

The plane determined by T and N is called the osculating plane of C at P. It is the plane that comes closest to approximating the curve near P.

Ex.:

Find the equations of the normal ~~tan~~ plane and osculating plane to the helix

$$r(t) = \langle \cos t, \sin t, t \rangle$$

at the point $P(0, 1, \pi/2)$.

normal plane: normal vector : $r'(\pi/2) = \langle -1, 0, 1 \rangle$

point : $(0, 1, \pi/2)$.

$$-1 \cdot (x-0) + 0 \cdot (y-1) + 1 \cdot (z - \pi/2) = 0.$$

Osculating ~~circle~~ plane:

point: $(0, 1, \pi/2)$

normal vector: binormal vector

⋮

~~point~~

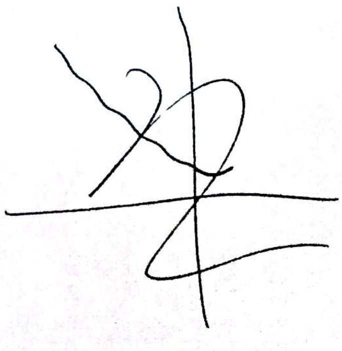
The osculating ~~circle~~ circle at P is the ~~circle~~ circle that best approximates the curve near P. ~~It~~ this circle lies on the osculating plane, ~~and then~~ lies on the concave side of the curve (normal direction), and has radius $1/k$.

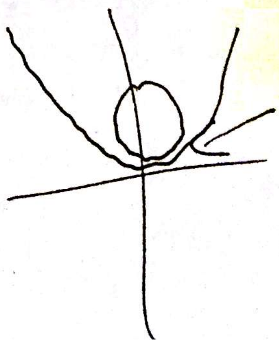
It shares the same tangent and normal vector and curvature.

Ex

Find and graph the osculating circle of the parabola $y=x^2$ at the origin.

The curvature is $k=2$. So the circle has radius $1/2$ and is centered at $(0, 1/2)$.





pretend that touches.

In general, the circle has radius $\frac{Y}{K}$ and has center $P + k \cdot N$.

in 3d, a circle also has inclination.