10.3 Polar Coordinates

A coordinate system is a method of specifying a point by an ordered list of numbers.

The cartesian coordinates specifies a point by giving the distance from the ~~x~~ x-axis and y-axis.

Polar coordinates

~~We~~ we start by choosing a pole (origin) and a polar axis (positive x-ray).

If P is some point on the plane, then let r be its distance from the pole and θ be the angle formed ~~with the polar axis~~. by OP and the polar axis. The polar representation of P is (r, θ).

~~Always~~
~~We'll start and think of the~~
~~Every point~~
~~We~~
~~Let~~

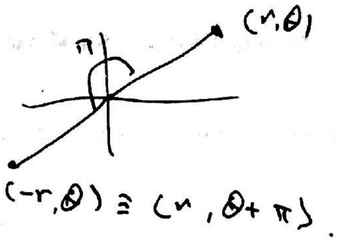
Every point in ~~the~~ a plane lies on some circle centered at the ~~o~~ origin. Every point on a circle can be specified by an angle.

We extend the definition of polar coordinates to allow negative radius. We set

≡ is made up notation.

→ $(-r, \theta) \equiv (r, \theta + \pi)$.

[just go in opposite direction]



Ex.

Plot the following:

(a) $(1, 5\pi/4)$

(b) $(2, 3\pi)$

(c) $(2, -2\pi/3)$

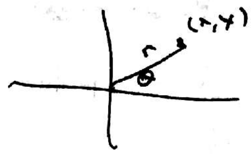
(d) $(-3, 3\pi/4)$.

Q. Give infinitely many representations of (r, θ).

$(r, \theta + 2n\pi)$,

$(-r, \theta + (2n+1)\pi)$.

Conversions with cartesian.



Cartesian → polar

$(x, y) \longmapsto \left(\sqrt{x^2+y^2}, \tan^{-1}\frac{y}{x}\right)$

polar → polar

$(r, \theta) \longmapsto (r\cos\theta, r\sin\theta)$.

Polar Curves

~~The graph of a polar equation $r = f(\theta)$ (or~~

~~A polar function~~

The graph of a polar equation $r = f(\theta)$ (or more generally $F(r,\theta) = 0$) is the set of points P that have at least one polar representation (r,θ) that satisfies the equation.

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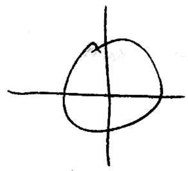
ex Graph the following and give Cartesian coordinates.

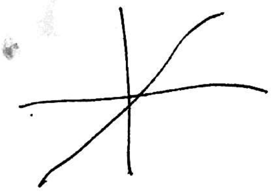
$r = 2$.

$x^2 + y^2 = 4$



$\theta = \pi/4$

$y = x$.



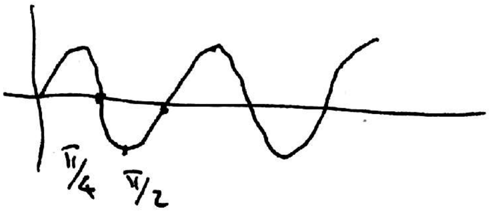
$r = 2\cos\theta$

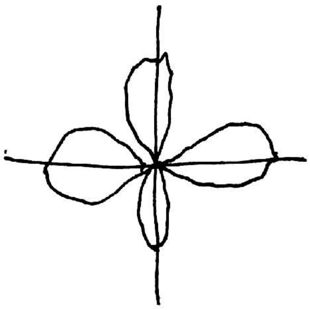
θ	$r = 2\cos\theta$
0	2
$\pi/6$	6
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0

θ	$r = 2\cos\theta$
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	-2

$r = \cos 2\theta$

$y = \cos 2x$
Cartesian





Symmetry

If the polar equation is unchanged when replacing θ by $-\theta$, then the curve is symmetric about the polar axis.

r replaced by $-r$

or θ replaced by $\theta + \pi$ (symmetric w/ rotation)

θ replaced by $\pi - \theta$

symmetric about $\theta = \pi/2$.

Tangents to polar curves:

· To find tangent line to polar curve $r = f(\theta)$, we regard θ as a parameter.

$x = r\cos\theta = f(\theta)\cos\theta$

$y = r\sin\theta = f(\theta)\sin\theta$.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dr}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{dx}\cos\theta - r\sin\theta}.$$

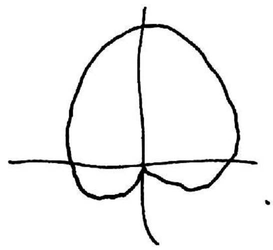
Ex

~~cardioidal~~

~~[illegible]~~ cardioid

Find the tangent line to the cardiod $r = 1+\sin\theta$

at $\theta = \pi/3$.



Ex

$$(x^2+y^2)^3 = 4x^2y^2$$