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Equation of sphere.

Derive from distance formula.

direction magnitude.

$$a = \frac{a}{|a|} \cdot |a|$$

12.3 Dot Product

The dot product ~~takes~~ to 2 vectors and returns a number related to the angle.

Defn If $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$, then the dot product ~~ais~~ of a and b is

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3.$$

(~~do~~ compute ~~each~~ component-wise multiplication then add them up.)

Properties.

If a, b, c are vectors and r is a scalar, then

1) $a \cdot a = |a|^2$ 2) $a \cdot b = b \cdot a$

3) $a \cdot (b + c) = a \cdot b + a \cdot c$ 4) $(ra) \cdot b = r(a \cdot b) = a \cdot (rb)$

5) $0 \cdot a = 0$.

Dot product as ~~an~~ information about angles.

Thm If θ is the ~~b~~ angle between a and b, then

$$a \cdot b = |a| \cdot |b| \cos\theta.$$

$$\Leftrightarrow \theta = \cos^{-1}\left(\frac{a \cdot b}{|a| \cdot |b|}\right) = \cos^{-1}\left(\left(\frac{a}{|a|}\right) \cdot \left(\frac{b}{|b|}\right)\right)$$

so angle $\Leftrightarrow$ dot product.

when are 2 vectors orthogonal?

perpendicular.

Thm ~~vectors~~ a and b are orthogonal iff $a \cdot b = 0$,

Examples

Find angle between $a = \langle 2, 2, -1 \rangle$ and

~~$b = \langle 5, -3, 2 \rangle$.~~

$b = \langle 4, -4, 2 \rangle$.

⋮

Find a unit vector orthogonal to both

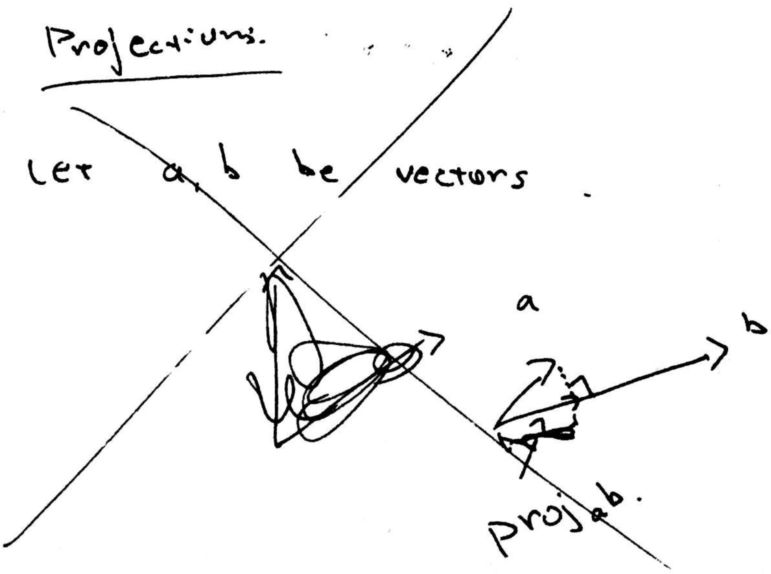
$a = \langle 3, 4, 5 \rangle$ and $b = \langle -1, 6, 7 \rangle$.

just set up.

~~vector~~

Projections.

Let a, b be vectors.

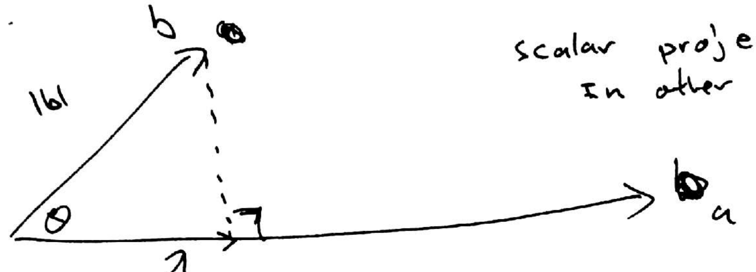


<u>Defn</u> Let a, b be vectors. Define $\text{proj}_a b$ to be the vector projection of b onto a.
Define $\text{comp}_a b$ to be the scalar projection of b onto a.
In other words,

$\text{comp}_a b = |\text{proj}_a b|$

only if angle is acute

ako show negative



proj a b

only works if angle is acute

magnitude: $|\text{proj}_a b| = \cos\theta \cdot |b| = \dfrac{a \cdot b}{|a|}$.

direction: $\dfrac{a}{|a|}$.

$\Rightarrow \text{proj}_a b = \dfrac{a \cdot b}{|a|} \cdot \dfrac{a}{|a|} = \dfrac{(a \cdot b)}{|a|^2} \cdot a$.

Does not depends on the length of a, only the direction.

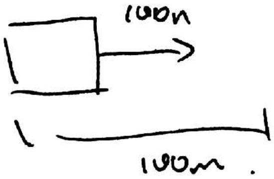
ex. $\text{Proj}_a b = \text{proj}_{2a} b$.

Let $V = \langle 3, 4 \rangle$ be your velocity vector
How fast are you going east?
How fast are you going along $\langle 1, 1 \rangle$?

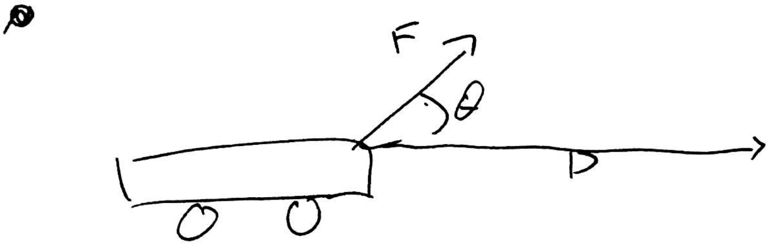
what is $\text{proj}_i \langle 4, 6 \rangle$?
$\text{comp}_{i+j} \langle 3, 4, 0 \rangle$?

<u>Work</u>

work is force · distance. ~~This only works when the force vector is in the same direction as displacement vector~~.



Then work is 100^2 n·m.



~~Th~~ Here work is defined to be the component of ~~force~~ F along ~~D~~ D. times the magnitude of D.

$$W = \text{comp}_D F \cdot |D|$$

$$= \frac{F \cdot D}{|D|} \cdot |D|.$$

Cross product and determinants.

Finding orthogonal vectors.

12.4 crossed product

The crossed product takes 2 vectors and returns an orthogonal vector.

Defi,

If $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$, then the crossed product of ~~ab~~ a and b is

$$a \times b = \langle a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1 \rangle.$$

~~3d deter~~
review 2d determinants

review 3d determinants