12.4 Cross Product

Defn Let $a = \langle a_1, a_2, a_3 \rangle$, $b = \langle b_1, b_2, b_3 \rangle$ be vectors in $\mathbb{R}^3$. Then the cross product

$$a \times b = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3 - a_1 b_2 - a_2 b_1 \rangle.$$

It is also a vector

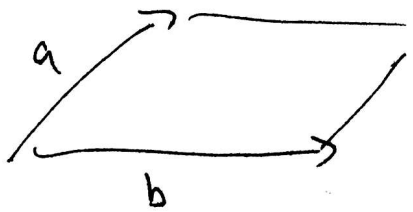
1) orthogonal to a and b, (2 directions)
2) whose direction is determined by the right-hand rule.
3) whose magnitude is $|a||b| \sin\theta$

compute $\langle 1, 2, 0 \rangle \times \langle 2, 4, -1 \rangle$ and $i \times j$, $j \times k$, $j \times i$.

Two nonzero vectors a and b are parallel iff

$$a \times b = 0.$$

the length of $a \times b$ is the area of the parallelogram determined by a and b.



Properties

See book.

key ones $a, b \in \mathbb{R}^3$, $c \in \mathbb{R}$

$a \times b = -b \times a$ / $c(a \times b) = (ca) \times b = a \times (cb)$.

$a \times 0 = 0$

Torque is

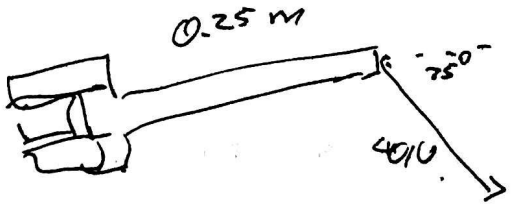
$\tau = r \times F$. talk about RH rule.

$|\tau| = |r \times F| = |r||F| \sin\theta$.

"Stranger" when $\theta = \pi/2$.

Ex

A bolt is tightened by applying a 40N force to a 0.25 m wrench as shown below



what is the magnitude of the torque?

$|\tau| = |r \times F|$

$= |r||F| \sin\theta$

$= 0.25 \cdot 40 \cdot \sin 75^\circ$.

12.5 Equation of lines and planes

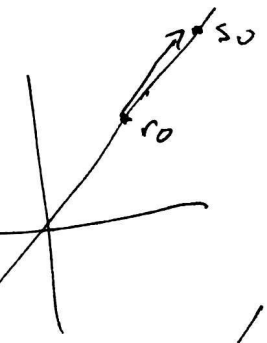
Lines

~~A line can be defined with the following~~

The defining data for a line can be:

take one point and the vector formed by taking difference.

{ two points } ⟷ { a point and a direction }



distinct

{ two points } ⟷ { a point and a direction. }

r_0, s_0 ⟷ r_0, $v \neq 0$

$r_0 \to r_0$, $r_0 \to v$, $s_0 - r_0$

$a \to a$

$a + x \to b$

$r_0 \to r_0$

$r_0 + v_0 \to s$

~~The vector~~

Then

Let L be a line. Then the vector equation for L is

$$r = r_0 + tv$$

Writing this equation out, we have

$$\underbrace{\langle x, y, z \rangle}_{r} = \underbrace{\langle x_0, y_0, z_0 \rangle}_{r_0} + t \underbrace{\langle a, b, c \rangle}_{v}$$

$$= \langle x_0 + at, y_0 + bt, z_0 + ct \rangle.$$

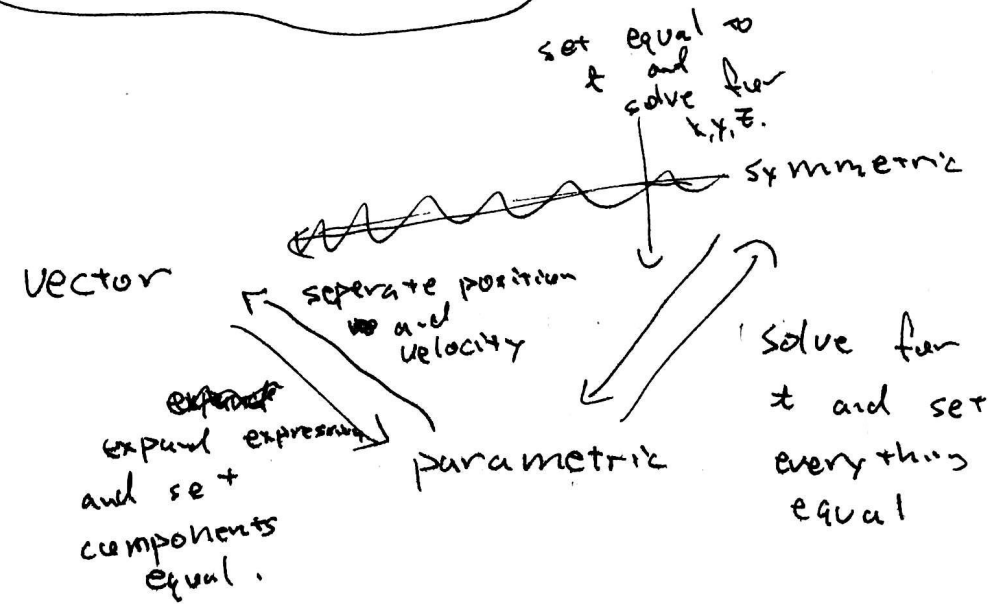
By equating components, we obtain the

parametric equations for L

$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct.$$

By solving for t, we obtain the symmetric equations for L,

$$t = \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$



Skew : does not intersect, not same direction

parallel : same direction

intersecting : intersects.

Let L_1 : $x = 1 + t$, $y = -2 + 3t$, $z = 4 - t$

L_2 : $x = 2s$, $y = 3 + s$, $z = -3 + 4s$.

Do conversions.

skew? Parallel? Intersect?