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The line segment from r_0 to r_1 is given by

$$r(t) = (1-t)\,r_0 + t\,r_1 \quad , \quad 0 \le t \le 1.$$

weighted average.

12.5 Equations of lines and planes

Planes

Idea

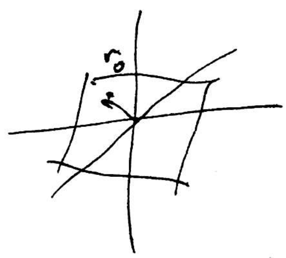
A plane can be given by a normal vector and a point.

"slope" $\quad n \cdot (r - r_0) = 0$

translate

$n \cdot r = n \cdot r_0$

vector equation of the plane.



$n \cdot (r - r_0) = 0$

$\Rightarrow \langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z_0 - z_0 \rangle = 0$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Scalar equation of the plane through (x_0, y_0, z_0) with normal vector $\langle a, b, c \rangle$.

$n \cdot r = n \cdot r_0$

$\Rightarrow \quad ax + by + cz = d$

linear equation

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The goal of any plane equation problem is to find a normal vector and a point.

Example

Find the equation of the plane that passes through $(1,3,2)$, $(3,-1,6)$, and $(5,2,0)$.

⋮

※ The angle between two planes is the angle between ~~this~~ their normal vectors (compute it with dot products).

Two planes are parallel if their normal vectors are parallel.

The distance between (x_1, y_1, z_1) and $ax+by+cz+d=0$ is

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}.$$

Derive ~~with~~ ~~...~~ ~~or w~~

Derive with derivatives or dot product.

12.6 Cylinders and Quadratic surfaces

Cylinders

~~A cylinder is a surface that consists of a lines (called rulings) that are parallel to a give~~

The trace of surface ~~with~~ against a plane is the intersection of ~~of~~ that surface and the plane.

Ex.

What is the trace of

$$x^2+y^2=1$$

against $z=2$? a circle

~~against $z=y$?~~

against $z=y$? a ellipse.

A cylinder is a surface whose trace along a line is constant. Up to rotation (the only case consider in the class), we can assume it is missing some variable in its equation.

Examples

$z = y^2 + 1$, parabolic cylinder

$z = x$. linear cylinder a.k.a. plane.

A quadric surface is the ~~graph~~ solutions to a quadratic equation.

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + \cdots + J = 0,$$

where some degree 2 term is nonzero.

After translation and rotation (only case considered in the class).

$$Ax^2 + By^2 + Cz^2 + J = 0$$

or $Ax^2 + By^2 + Iz = 0$.

and all the permutations.

Examples

All quadratic cylinders.

traces

$z = y^2 + 1$.

$x = k$,

$y = k$,

$z = k$.

$$x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

Ellipsoid

traces.

$$\cancel{\frac{x^2}{4}} + y^2$$

$$z = 4x^2 + y^2$$

elliptic paraboloid.

Traces.

$$z = y^2 - x^2$$

hyperbolic paraboloid

traces.