12.6 Cylinders and Quadratic surface.

If we start ~~Starting~~ with the "graph" of $f(x,y,z)=0$, then

- $f(ax,y,z)=0$ shrinks the graph in the x-direction by a factor of a.
- $f(x,y/a,z)=0$ stretch the graph in the y-direction by a factor of a
- $f(x,y-a,z)=0$ shifts in the positive y-direction by a.
- $f(x,y,-z)=0$ reflects across the $z=0$ plane.

Ex

$x^2+y^2+z^2=1$ is a sphere of radius 1.

$\Rightarrow \dfrac{x^2}{2^2}+\dfrac{y^2}{3^2}+\dfrac{z^2}{4^2}$ is a ellipsoid. that is bounded by $x=\pm 2$, $y=\pm 3$, $z=\pm 4$.

Read ~~the~~ Table 1 of Chapter 12.6. things are in standard form.

Defn The standard form of a quadratic surface equation is an equation for the quadratic surface such that

- if a X^2 term appears then X does not. (1)
- the constant term is either 0 or 1. (2)
- group linear terms with constant term. (3)

ex

$z-2=\dfrac{x^2}{5^2}+(y-1)^2$ elliptic paraboloid, with vertex $(0,1,2)$.

Tech

How to put a equation into standard form?

- completing the square to handle (1)
- dividing by constant term. to handle (2).

Do (1) then (2).

ex

classify the quadratic surface

$$x^2 + 2z^2 - 6x - y + 10 = 0.$$

$$(x^2 - 6x) - y + 2z^2 + 10 = 0$$

$$\Rightarrow (x-3)^2 - 9 - y + 2z^2 + 10 = 0$$

$$\Rightarrow (x-3)^2 - y + 2z^2 + 1 = 0$$

$$\Rightarrow (x-3)^2 + 2z^2 = y - 1$$

elliptic paraboloid with vertex

$(3, 1, 0)$.

~~where the ...~~

that is bigger in the z direction.
stretch

Tech

Suppose a problem asks to determine the equation of a hyperboloid of one sheet that fits some description. If we know it is centered at the origin then we know it is of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1.$$

we have 3 variables a, b, c. so we need 3 equations.

10.1, 13.1 Vector functions and space curves

Defn

A vector function is a function

$$r : \mathbb{R} \to \mathbb{R}^n$$

$$r(t) = \langle f(t), g(t), h(t) \rangle.$$

parameter

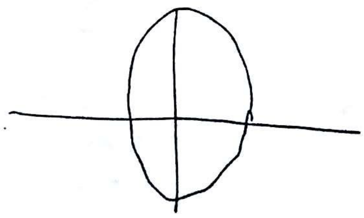
component functions

The image of a vector function is a space curve.

the set of points in the range.

Example

$$r(t) = \langle \cos(t), 2\sin(t) \rangle$$



same space curve as

$$s(t) = \langle \cos(2t-1), 2\sin(2t-1) \rangle.$$

Defn

If $r(t) = \langle f(t), g(t), h(t) \rangle$. Then

$$\lim_{t \to a} r(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$$

provided the limits of the component functions exist.

Defn

A vector function r is continuous at a if

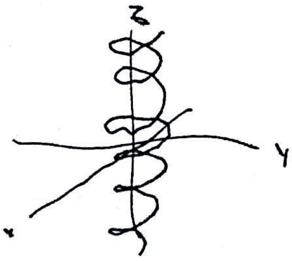
$$\lim_{t \to a} r(t) = r(a)$$

Example

~~What is a vect~~

Give a vector function whose space curve is a helix.

$r(t) = \langle \sin(t), \cos(t), t \rangle$.



Find the vector function that parameterizes the intersection of the cylinder $x^2 + y^2 = 1$ (1) and the plane (2) $y + z = 2$.

(1) $x = \cos t$
$y = \sin t$

(2) $z = 2 - \sin t$.