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10.2 Calculus with parametric curves

Tangents

Suppose $x = f(t)$ and $y = g(t)$. Then by chain rule, we have

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}.$$

If $dx/dt \neq 0$, we can solve for $\frac{dy}{dx}$:

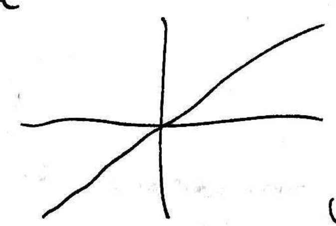
$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} \quad \text{if } \frac{dx}{dt} \neq 0.$$

Similar for $\frac{dx}{dy}$.

The book lies. This is not the full story.

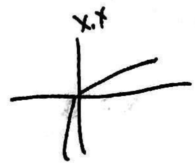
For example, if
$x = t^3$
$y = t^3$

then



can show with ~~carful~~ l'hopital.

Here $\frac{dx}{dt} = \frac{dy}{dt} = 0$, but $\frac{dy}{dx} = 1$. And if

$x = y =$ 

then $\frac{dx}{dt}$ and $\frac{dy}{dt}$ DNE but $\frac{dy}{dx} = 1$!

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}.$$

Perhaps not what you expect so be careful.

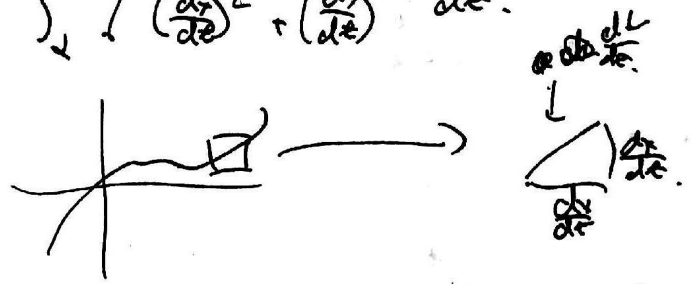
Arc length

If $y = F(x)$ and F' is continuous, then the arc length of the curve formed by (x, y) with $a \le x \le b$ is

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

If $x = f(t)$ and $y = g(t)$, then the arc of of the curve formed by (x, y) with $\alpha \le t \le \beta$ is

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$



if the curve is traversed by exactly once ?!

For example, $x = 0$, $y = \cos t$.

a counter

13.2 Derivatives and Integral of ~~...~~ vector functions

$\langle f, g, h \rangle$

The derivative r' of a vector function r is defined to be

$$\frac{dr}{dt} = r'(t) = \lim_{h \to 0} \frac{r(t+h) - r(t)}{h}$$

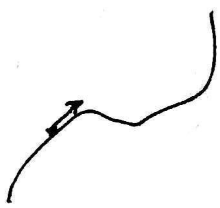
$$= \left\langle \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}, \lim \frac{g(t+h) - \dots}{}, \dots \right\rangle$$

$$= \langle f'(t), g'(t), h'(t) \rangle .$$

~~Then~~

Now suppose $r'(t)$ is nonzero. Then the unit tangent vector

$$T(t) = \frac{r'(t)}{|r'(t)|} .$$



~~There exists~~

The tangent line of the curve defined by $r(t)$ through a point P on the curve is defined to be the line with ~~to~~ tangent vector $T(t)$ (or $r'(t)$) and that passes through P.

$$\int_a^b r(t)\,dt = \left\langle \int_a^b f(t)\,dt, \int_a^b g(t)\,dt, \int_a^b h(t)\,dt \right\rangle.$$

If r is velocity, then $\int_a^b r(t)\,dt$ is displacement.

Example. All things with helix.