Solution

1. (12 points)

A B C

(a) Find the equation of the plane that goes through the three points (1, 2, 5), (2, 2, 2), (3, 3, 3).

$\vec{AB} = \langle 1, 0, -3 \rangle$

$\vec{AC} = \langle 2, 1, -2 \rangle$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -3 \\ 2 & 1 & -2 \end{vmatrix} = (0 - -3)\vec{i} - (-2 - -6)\vec{j} + (1-0)\vec{k} = \langle 3, -4, 1 \rangle$$ ← check dot products ✓

NORMAL

$$\boxed{\begin{array}{c} 3(x-1) - 4(y-2) + (z-5) = 0 \\ 3x - 3 - 4y + 8 + z - 5 = 0 \\ 3x - 4y + z = 0 \end{array}}$$

ANY POINT

$\vec{n_1} = \langle 1, 0, -1 \rangle$

(b) Find parametric equations for the line of intersection of $x - z = 10$ ① and $x + y + 2z = 0$ ②.

$\vec{n_2} = \langle 1, 1, 2 \rangle$

Sol. #1: FIND TWO POINTS: $x = 0 \Rightarrow z = -10$ in ①

① & ② $\Rightarrow (0) + y + 2(-10) = 0 \Rightarrow y = 20$

$P(0, 20, -10)$

$z = 0 \Rightarrow x = 10$ in ①

① & ② $\Rightarrow (10) + y + 2(0) = 0 \Rightarrow y = -10$

$Q(10, -10, 0)$

DIRECTION: $\vec{v} = \vec{PQ} = \langle 10, -30, 10 \rangle$

$$\boxed{\begin{array}{l} x = 0 + 10t \\ y = 20 - 30t \\ z = -10 + 10t \end{array}}$$

ANY POINT ON LINE

ANY VECTOR PARALLEL TO $\langle 10, -30, 10 \rangle$

Sol. #2:

- Find one point. (ex: (0,20,-1
- Find direction vector:

$\vec{v} = \vec{n_1} \times \vec{n_2}$

Since both $\vec{n_1}$, $\vec{n_2}$ are perpendicular to $\vec{v}$