Plan

- 1.3
- 1.4

1.3

- Definition: A subset W of a vector space V over a field F is called a subspace of V if W is a vector space over F with the operations of addition and scalar multiplication defined on V.
- Problem: Normally, there are 8 properties you need to check. But it turns out you only need to check 4 of them. Which 4? Why?
- Problem: (Theorem 1.3) Let V be a vector space and W and subset of V. Then W is a subspace of V if and only if the following three conditions hold for the operations defined in V.

  1. $0 \in W$.
  2. $x + y \in W$ whenever $x \in W$ and $y \in W$.
  3. $cx \in W$ whenever $c \in F$ and $x \in W$.

- Problem: Same problem as before, but replace conditions 2 and 3 with

  - $cx + y \in W$ whenever, $x, y \in W$ and $c \in F$.

- Problem: Give an example of a vector space V and a subset W of V such that, W is a vector space but W is not a subspace of V.
- Problem: Show that the intersection of 2 subspaces is a subspace.

1.4

- Definition: Let V be a vector space and S a nonempty subset of V. A vector $v \in V$ is called a linear combination of vectors of S if there exists a finite number of vectors $u_1, \ldots, u_n$ in S and scalars $a_1, \ldots, a_n$ in F such that $v = a_1 u_1 + \cdots + a_n u_n$.
- Problem: We denote the set of all linear combinations of S by span$S$. By convention, we define the span of the empty set to be the trivial subspace $\{0\}$. Prove that span(S) is always a subspace.
- Problem: Let $S \subseteq T$ be sets inside of a vector space V. Prove that span(S) is a subspace of span(T).
- Problem: Prove that span(S) is a the smallest subspace containing S. (This gives an alternative definition of span(S) that turns out to be quite useful!)