Jan 24

- Introduce nullspace and columns space
- Draw picture of domain, range, kernel, codomain

Theorem related to linear independence

Let $S = \{a_1, \ldots, a_n\} \subseteq \mathbb{R}^m$ be a set of vectors. Let A be the $m \times n$ matrix formed by writing the elements of S as columns. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be the linear transformation defined by $T(x) = Ax$.

- S is linearly independent
- $Ax = 0$ has only the trivial solution
- For any b, $Ax = b$ has either no solution or exactly one solution.
- $null(A) = \{0\}$
- T is one-to-one
- $ker(T) = \{0\}$

Theorem related to spanning

Let $S = \{a_1, \ldots, a_n\} \subseteq \mathbb{R}^m$ be a set of vectors. Let A be the $m \times n$ matrix formed by writing the elements of S as columns. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be the linear transformation defined by $T(x) = Ax$.

- S is spanning
- $Ax = b$ has a solution for any b
- $col(A) = \mathbb{R}^m$
- T is onto
- $range(T) = \mathbb{R}^m$

Theorem related to the square case

Let $S = \{a_1, \ldots, a_n\} \subseteq \mathbb{R}^n$ be a set of vectors. Let A be the $n \times n$ matrix formed by writing the elements of S as columns. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be the linear transformation defined by $T(x) = Ax$.

- S is a basis
- S is linearly independent
- S is spanning
- $Ax = b$ always has a unique solution
- $col(A) = \mathbb{R}^n$
- $null(A) = \mathbb{R}^n$
- T is invertible
- A is invertible

3.2 Matrix Algebra

Matrix multiplication is weird

- $AB \neq BA$
- Explain what $AB = 0$ means in terms of columnspace and nullspace

Tranpose of a matrix

- Teach how to tranpose
- $(A + B)^t = A^t + B^t$
- $(sA)^t$
- $(AC)^t = C^t A^t$

Diagonal matrices and upper triangular matrices is a thing

- Give definition
- The product of digaonl is diagonal. Discuss the effects of multiplying a matrice by a diagonal matrix
- The product of upper triangulars is upper triangular

Powers of matrices is a thing

- Powers of diagonal is easy
- Wouldn't it be great if $A = UDU^{-1}$

3.3 Inverses

- Explain what an inverse is.
- Derive inverse formula.