Counting Principles

Suppose there are 2 red and 6 blue marbles in a jar. If you sample two, what is the probability of getting no red?

In [1]:
```
(6/8)*(5/7)
```

Out[1]: 15/28

Of getting all red?

In [2]:
```
(2/8)*(1/7)
```

Out[2]: 1/28

Of getting one red and one blue? Multiply by two because blue and red can occur in either order.

In [3]:
```
(2/8)*(6/7)*2
```

Out[3]: 3/7

Combinations

You can also calculate the probablity by finding the total number of possible successes divided by the total number of possibilities.

The number of ways that two blue marbles can be selected is C(6, 2) or ${}_6C_2$, and the total number of ways that any two marbles can be selected is C(8, 2), so the probability of selecting two blue is:

In [4]:
```
binomial(6,2)/binomial(8,2)
```

Out[4]: 15/28

For selecting two red:

In [5]:
```
binomial(2,2)/binomial(8,2)
```

Out[5]: 1/28

For selecting one red and one blue, the red and blue selections are independent, so the probabilities are multiplied:

In [6]:
```
binomial(2,1)*binomial(6,1)/binomial(8,2)
```

Out[6]: 3/7

For small samples either counting or combinations work equally well and give the same answer. For larger samples and populations, combinations are much easier.

Suppose there are 6 defective regrigerators and 44 functional in a shipment. What is the probability of selecting three or more defective refrigerators in a sample of five?

The probability of selecting three defective is:

In [7]:
```
Pr_3 = binomial(6,3)*binomial(44,2)/binomial(50,5); Pr_3
```

Out[7]: 473/52969

This is approximately

In [8]:
```
Pr_3.n(digits=5)
```

Out[8]: 0.0089297

Similarly the probability of four or five in the selection can be calculuated.

In [9]:
```
Pr_4 = binomial(6,4)*binomial(44,1)/binomial(50,5); Pr_4
Pr_5 = binomial(6,5)*binomial(44,0)/binomial(50,5); Pr_5
print(Pr_4, Pr_5)
```

```
33/105938 3/1059380
```

The probability of three or greater is just the sum.

In [10]:
```
total = Pr_3 + Pr_4 + Pr_5
print(total, total.n(digits=5))
```

```
1399/151340 0.0092441
```

In [11]:
```
# using combinations again, the probability of zero defective in five trials
P = binomial(6,0) * binomial(44, 5) / binomial(50, 5)
print(P, P.n(digits=5))
```

```
19393/37835 0.51257
```

In [12]:
```
# and probability of one defective in five trials
P = binomial(6, 1) * binomial(44, 4) / binomial(50, 5)
print(P, P.n(digits=5))
```

```
58179/151340 0.38443
```

In [13]:
```
# and probability of one defective in one trial
P = binomial(6, 1) * binomial(44, 0) / binomial(50, 1)
print(P, P.n(digits=5))
```

```
3/25 0.12000
```

Binomial Distribution

For an event with probability P, the probability that it will occur r times in n trials is:

$$Pr(n,r) = \binom{n}{r} P^r (1-P)^{n-r}$$

This works if the probability is constant.

In [14]:
```
# probability of the event, note the similarity to the above problem
P = 6 / 50
```

In [15]:
```
# the function
Pr(n, r) = binomial(n,r) * P^r * (1-P)^(n-r)
```

In [16]:
```
# the probability of an event happening once in five trials
print(Pr(5,1), Pr(5,1).n(digits=5))
```

```
702768/1953125 0.35982
```

In [17]:
```
# the probability of an event happening three times in five trials
print(Pr(5,3), Pr(5,3).n(digits=5))
```

```
26136/1953125 0.013382
```

In [18]:
```
# the probability that the event will happen three or more times in five tri
total = 0
for i in [3, 4, 5]:
    total += Pr(5, i)
print(total, total.n(digits=5))
```

```
139833/9765625 0.014319
```

In [0]:
```

```