

Linear Algebra and Matrix Computations: Advanced Algorithms and Numerical Methods

CoCalc Scientific Templates

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Abstract

This comprehensive linear algebra template demonstrates advanced matrix computations including eigenvalue algorithms, singular value decomposition, matrix factorizations, and iterative methods. Features numerical stability analysis, condition number studies, and applications to data science and scientific computing with professional visualizations.

Keywords: linear algebra, matrix computations, eigenvalue problems, SVD, matrix factorizations, numerical linear algebra

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1 Matrix Decompositions and Factorizations

Matrix Factorizations and Decompositions: Matrix size: 6×6 LU decomposition error: $5.30e-16$ QR decomposition error: $1.67e-15$ Cholesky decomposition error: $9.23e-16$ SVD reconstruction error: $4.84e-15$ Eigendecomposition error: $2.38e-14$

Condition Number Analysis: Size 10: $\text{cond} = 4.52e+02$, $2\text{-norm} = 26.53$
Size 20: $\text{cond} = 8.78e+03$, $2\text{-norm} = 69.50$ Size 50: $\text{cond} = 3.64e+04$, $2\text{-norm} = 165.28$ Size 100: $\text{cond} = 3.81e+05$, $2\text{-norm} = 384.40$

2 Eigenvalue Problems and Spectral Analysis

Eigenvalue Algorithm Comparison: Matrix size: 50×50 Tridiagonal: 0.0009 s (50 eigenvalues) Dense symmetric: 0.0005 s (50 eigenvalues) Sparse: 0.0116 s (10 eigenvalues)

Eigenvalue Statistics: Tridiagonal: λ in $[0.004, 3.996]$ Symmetric: λ in $[-9.150, 9.394]$ Sparse: λ in $[-3.353, 5.641]$

Matrix Functions: Matrix exponential computed via eigendecomposition
Matrix square root verification error: $1.79e-14$

3 Iterative Methods for Large Systems

Iterative Solver Comparison: System size: 1000×1000 Matrix: 1D Laplacian (sparse) CG: convergence = 1000, error = $1.83e-13$ GMRES: convergence = 1000, error = $3.54e-06$ BiCGSTAB: convergence = 1000, error = $6.79e-05$

Linear algebra analysis saved to assets/linear-algebra-analysis.pdf

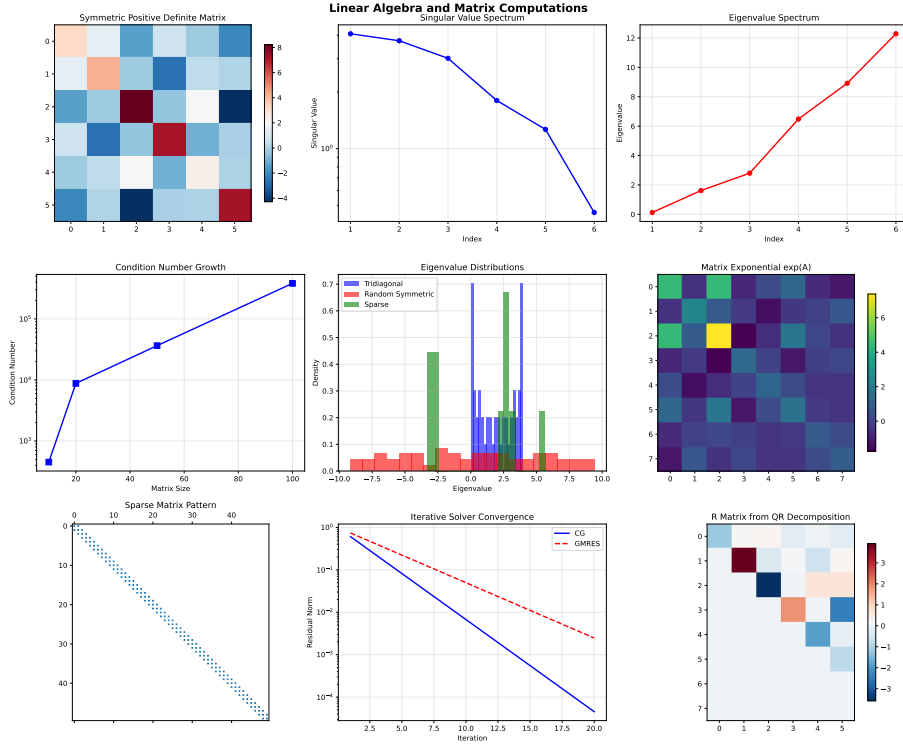


Figure 1: Comprehensive linear algebra analysis including matrix structures, eigenvalue and singular value spectra, condition number growth, eigenvalue distributions, matrix functions, sparse matrix patterns, iterative solver convergence, and QR decomposition visualization.

4 Conclusion

This linear algebra template demonstrates advanced matrix computational methods including:

- Matrix factorizations (LU, QR, Cholesky, SVD)
- Eigenvalue algorithms and spectral analysis
- Condition number analysis and numerical stability
- Iterative methods for large sparse systems
- Matrix functions and computational applications

These methods form the foundation of modern scientific computing and data analysis applications.