Problem solving with Chris
Week 3

Remember, it isn't the answer that counts, but the justification.

1. How many six digit numbers are there that have at least one even digit?

2. There are six letters in the Hermitian language. A word is any sequence of six letters, some pair of which are the same. How many words are there in the Hermitian language?

3. Can a number written with one hundred zeroes, one hundred ones and one hundred twos be a perfect square?

4. What is the largest number of kings which can be placed on a chessboard so that no two of them are in check?

5. What is the size of the largest subset of $\{1, 2, \ldots, 2n\}$ such that no element divides any other?

6. A polygon is cut out of paper, and then folded in two along a straight line. Prove that the perimeter of the polygon formed is not greater than the perimeter of the original polygon.

7. Two children take turns breaking up a rectangular chocolate bar six squares wide by eight squares long. They may break the bar only along the divisions between the squares. If a bar breaks into several pieces, they keep breaking the pieces until only the individual squares remain. The player who cannot make a break loses the game. Who will win?

8. There are several cities in a certain kingdom. An obnoxious citizen is exiled from city A to city B, which is the furthest city in the kingdom from A. After a while, he is exiled from city B to the furthest city from it, which happens to be different to A. Prove that, if his exiles continue in the same way, he will never return to city A.

9. (29th IMO. This problem is really cool) A function f is defined on the positive integers by

$$\begin{aligned} f(1) &= 1 \\ f(3) &= 3 \\ f(2n) &= f(n) \\ f(4n+1) &= 2f(2n+1) - f(n) \\ f(4n+3) &= 3f(2n+1) - 2f(n) \end{aligned}$$

for all positive integers n. Determine the number of positive integers n, less than or equal to 1988, for which $f(n) = n$.