PROBABILITY - PROBLEM SET 4

MATHCAMP 1999

1. Suppose two numbers are chosen independently, at random, from between 0 and 1. Each of the following questions are separate (meaning: do not use statements in previous questions to answer other questions)
   (a) What's the probability their sum is greater than .75?
   (b) If I tell you their sum is greater than .5, now what is the probability their sum is greater than .75?
   (c) If I tell you the second number is greater than .5, now what tis the probability their sum is greater than .75?
   (d) What's the probability that the sum of the squares of the two numbers is less than 1?

2. We're going to play a common carnival game. It involves tossing pennies (which are about .75 inches in diameter). We're tossing the penny at some kind of uniform mesh - we win if the penny doesn't touch any of the mesh lines, lose if the penny touches any line.
   (a) If the mesh is of 1 inch squares (assume the lines are infinitely thin), what's the probability of winning?
   (b) Suppose the area of each grid box is kept the same (one square inch), but the dimensions are changed. What happens to the probability of winning?
   (c) Suppose the grid is changed to equilateral triangles with area of one square inch, what's the probability of winning now?
   (d) Suppose the grid is simply parallel lines, spaced one inch apart, what's the probability of winning now?

3. If a chord is selected at random on a fixed circle, what is the probability that its length exceeds the radius of the circle?

4. Suppose three points are picked at random on a circle. What's the probability that the triangle with those three points as vertices contains the center of the circle?

5. Duels in the town of Discretion are rarely fatal. There, each contestant comes at a random moment between 5 and 6 a.m. on the appointed day and leaves exactly 5 minutes later, honor served, unless his opponent arrives within the time interval and they fight. What fraction of duels lead to violence? What if the waiting time is lengthened to 10 minutes?

6. If a stick is broken at random, what is the expected length of the smaller piece?

7. What is the probability that the equation $x^2 + 2bx + c = 0$ has real roots? (First try for b and c picked randomly in a square that goes from -n to n. Then let n go to ∞.)

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