

All work on this lab should be the collective effort of all group members. Technology allowed on this lab includes: Desmos (<https://www.desmos.com/calculator>) and an approved TI calculator. This lab has 7 questions for a total of 8 points.

1. (**Lorentz contraction**) In relativity theory, the length of an object, say a rocket, appears to an observer to depend on the speed at which the object is traveling with respect to the observer. If the observer measures the rocket's length as L_0 at rest, then at speed v , the length will appear to be

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}.$$

This equation is the **Lorentz contraction** formula. Here, c is the speed of light in a vacuum, which is about 3×10^8 m/sec.

- (a) (4 points) What happens to L as v increases?

- (b) (4 points) Find $\lim_{v \rightarrow c^-} L$.

(b) _____

- (c) Why was the left-hand limit needed in the limit?

2. Let $f(x) = \sqrt{x+1}$. Find the largest $\delta > 0$ such that $f(x) \in (2 - \varepsilon, 2 + \varepsilon)$, when $\varepsilon = 0.1$.

3. Use epsilon-delta proofs of a limit to prove that $\lim_{x \rightarrow -11} \frac{x^2 + 6x + 5}{x + 5} = -10$.

4. Consider the function $g(x) = \begin{cases} 0, & x \leq 0 \\ \sin\left(\frac{1}{x}\right), & x > 0 \end{cases}$.

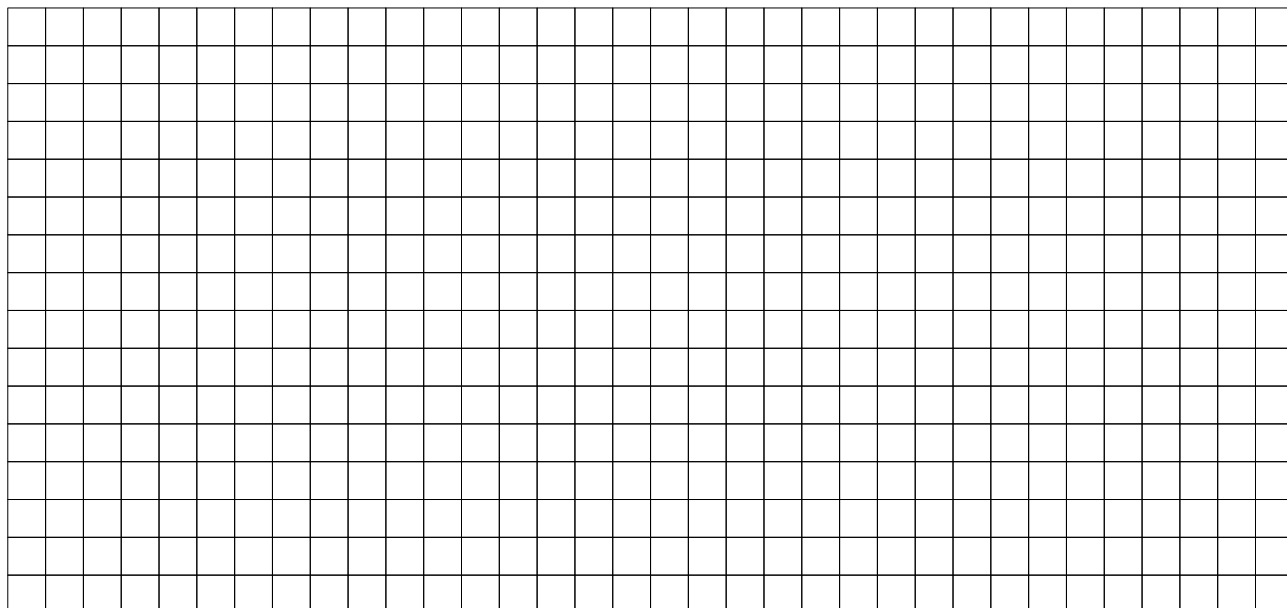
(a) Does $\lim_{x \rightarrow 0^+} g(x)$ exist? If so, what is it? If not, why not?

(b) Does $\lim_{x \rightarrow 0^-} g(x)$ exist? If so, what is it? If not, why not?

(c) Does $\lim_{x \rightarrow 0} g(x)$ exist? If so, what is it? If not, why not?

5. Graph a function that meets the following criteria:

- $\bullet \lim_{x \rightarrow -\infty} h(x) = -\infty,$
- $\bullet \lim_{x \rightarrow -1^+} h(x) = 2,$
- $\bullet h(3) = 0,$
- $\bullet \lim_{x \rightarrow -1^-} h(x) = -3,$
- $\bullet \lim_{x \rightarrow 3} h(x) = 4,$
- $\bullet \lim_{x \rightarrow \infty} h(x) = -1.$



6. Each of the following statements are false. Find a counterexample to each of the statements. Your counterexample may take many different forms, i.e. you may come up with an equation, provide a graph, or give a detailed explanation.

(a) For any function $f(x)$, $\lim_{x \rightarrow c} f(x) = f(c)$.

(b) $\lim_{x \rightarrow \infty} \sin(x) = 0$ because $\frac{-1+1}{2} = 0$.

(c) All limits of the form $\infty - \infty$ must go to zero.

(d) If $\lim_{x \rightarrow c} f(x) = L_1$ and $\lim_{x \rightarrow L_1} g(x) = L_2$, then $\lim_{x \rightarrow c} (g \circ f)(x) = L_2$.

7. Calculate the following limits using algebra.

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x}$

(b) $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{1 - x}$

(c) $\lim_{x \rightarrow \infty} \frac{x^7 - 11x^3 - 5x - 2}{3x - 2x^2 - 17x^{11} + 12}$

(d) $\lim_{x \rightarrow 0} \frac{\sin(7x)}{2x}$

(e) $\lim_{x \rightarrow 0} \frac{3e^x - 3}{3e^{2x} + 9e^x - 12}$