All work on this lab should be the collective effort of all group members. Technology allowed on this lab includes: Desmos (`https://www.desmos.com/calculator`) and an approved TI calculator. This lab has 8 questions for a total of 60 points.

1. (20 points) Determine if the following statments are true or false. You must provide a justification for your answer.

   (a) TRUE / FALSE

   If a number is divisible by 6, then it is divisible by 3.

   (b) TRUE / FALSE

   For all real numbers x and y, $\frac{x}{y} = 0$ if and only if $x = 0$.

   (c) TRUE / FALSE

   For all real numbers y there is a real number x such that $y = 2x + 4$.

   (d) TRUE / FALSE

   For all real numbers $x > 0$ and $y > 0$, if $x > y$, then $\dfrac{1}{x} < \dfrac{1}{y}$

2. Consider the following implication:

   If x is divisible by 12, then x is divisible by 3.

   (a) (2 points) Write the converse of the statement.

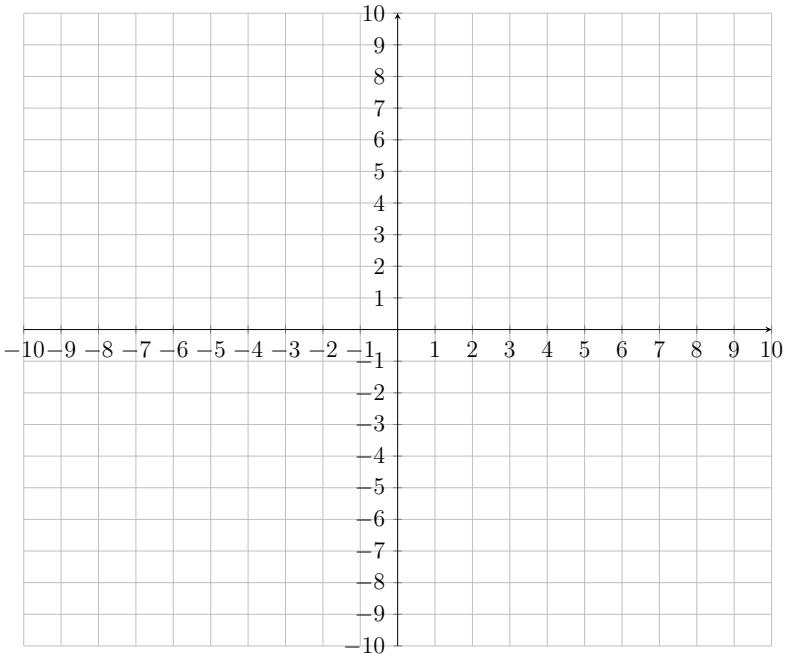
   The converse of the implication is TRUE / FALSE.

   (b) (2 points) Write the contrapositive of the statement.

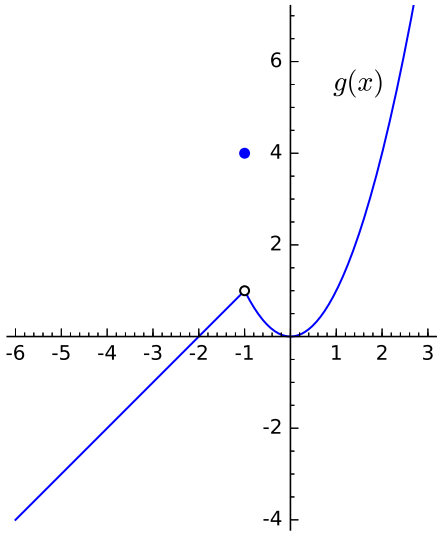
   The contrapositive of the implication is TRUE / FALSE.

3. (6 points) Sketch the graph of a function that has the given limits and values. There is more than one correct answer.

   - $\lim_{x \to -1} f(x) = 4$
   - $\lim_{x \to \infty} f(x) = 4$
   - $f(0) = 5$



4. (6 points) Let $g(x)$ be the function graphed below.



(a) $\lim_{x \to -1} g(x) =$ _____

(b) $\lim_{x \to 1} g(x) =$ _____

(c) $g(-1) =$ _____

(d) $\lim_{x \to 2} g(x) =$ _____

(e) $\lim_{x \to -\infty} g(x) =$ _____

(f) $\lim_{x \to \infty} g(x) =$ _____

5. (5 points) Give an argument, be it a table of values, or a graph, that justifies your educated guess for the following limit:

$$\lim_{x\to 4} \frac{x-4}{(x+1)(x-4)}.$$

6. (4 points) Let $h(x)$ be the following piecewise-defined function:

$$h(x) = \begin{cases} 3x-2 & \text{if } x > 2 \\ 0 & \text{if } x = 2 \\ x^2 & \text{if } x < 2 \end{cases}.$$

Using the graph of the function, determine the value of $\lim_{x\to 2} h(x)$.

6. _____

7. (5 points) Write the following limit using the $\varepsilon - \delta$ definition of a limit.

$$\lim_{x\to 3} \left(x^2 - 4\right) = 5$$

8. (10 points) Given that $\lim_{x\to 2} (2x+1) = 5$, use algebra to approximate the largest value of δ such that

$$\text{if } x \in (2-\delta, 2) \cup (2, 2+\delta), \text{ then } f(x) \in (5-\varepsilon, 5+\varepsilon) \text{ where } \varepsilon = 0.01.$$