Cubic Braid Groups

Talk on the Sage Days 94 in Saragossa

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following the introduction given on CoCalc

Welcome to the Cubic Braid Group page on CoCalc

This page contains a new class declaration to be used with sage. It deals with certain factor groups of the Artin braid group. This class is not integrated into the sage library, right now. You can test it here on CoCalc (if you are signed in to your own acount) or on your own computer (if you have got sage installed on it). Everyone is invited to help to improve this class. If you like to get full access to this project (as a collaborator) please contact "s.oehms@web.de".

To learn more about this new class you may follow one of the following links:

Introduction to the Cubic Braid Groups using jupyter notebook (.ipynb)

Introduction to the Cubic Braid Groups using sage worksheet (.sagews)

Download the introduction to the Cubic Braid Groups as PDF

Installation instruction:

Installation of the Cubic Braid Group class

Download

Go to the "Files"-Menue an click on the Icon on the right of "cbg.tgz" (cloud with download arrow) !

Overview

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Introduction

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$$s_i^3 = 1$$

In general these groups have firstly been investigated by Coxeter, H.S.M in: "Factor groups of the braid groups, Proceedings of the Fourth Candian Mathematical Congress (Vancover 1957), pp. 95-122".

Coxeter showed, that these groups are finite as long as the number of strands is less than 6 and infinite elsewise. More explicitely the factor group on three strand braids is isomorphic to $SL(2,3)$, on four strand braids to $GU(3,2)$ and on five strand braids to $Sp(4,3) \times C_3$. Coxeter realized these groups as subgroups of unitary groups with respect to a certain hermitian form over the complex numbers (in fact over $\mathbb{Q}$ adjoined with a primitive 12-th root of unity).

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sage: braid5_2 = B3((-1, -1, -1, -2, 1, -2))
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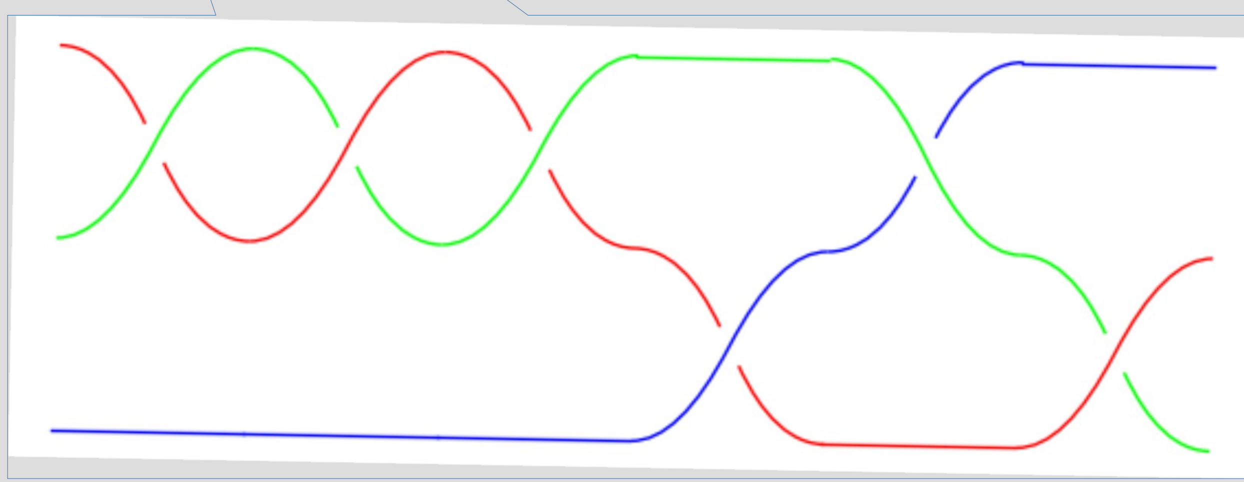

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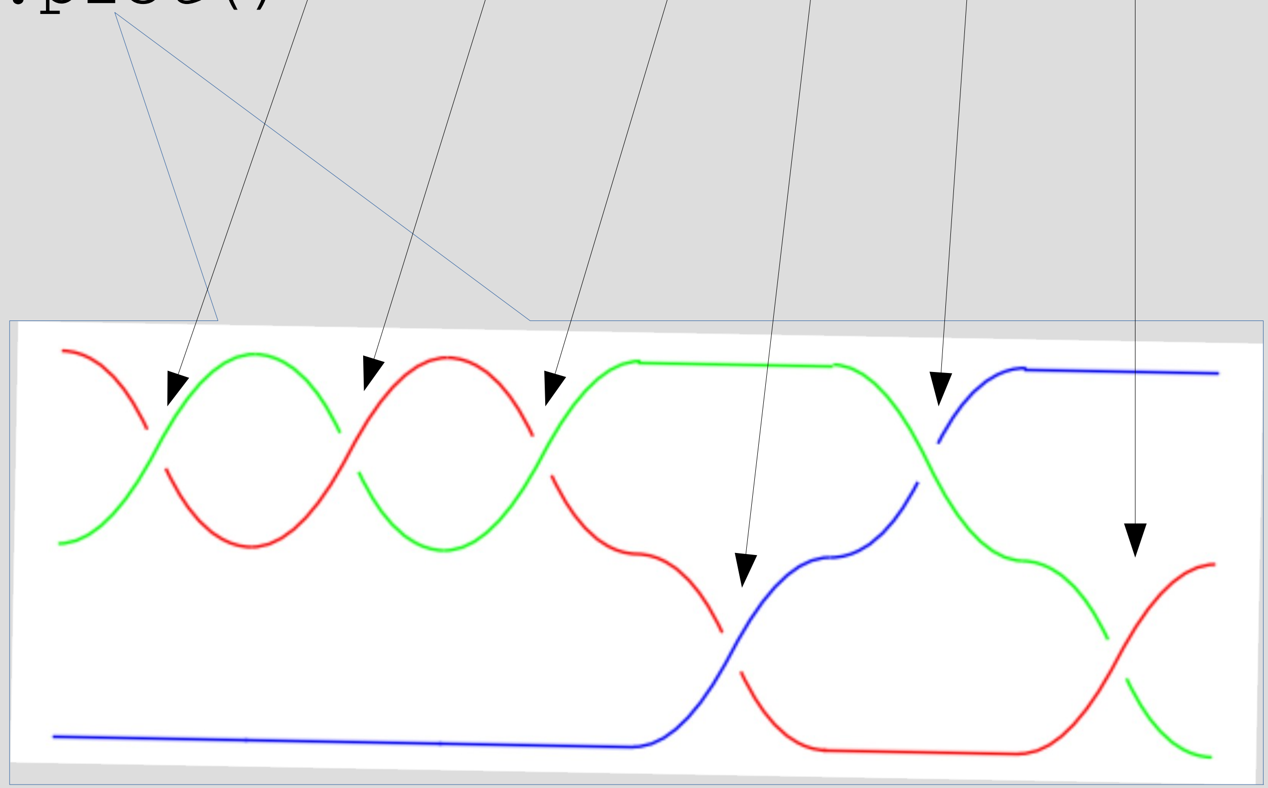

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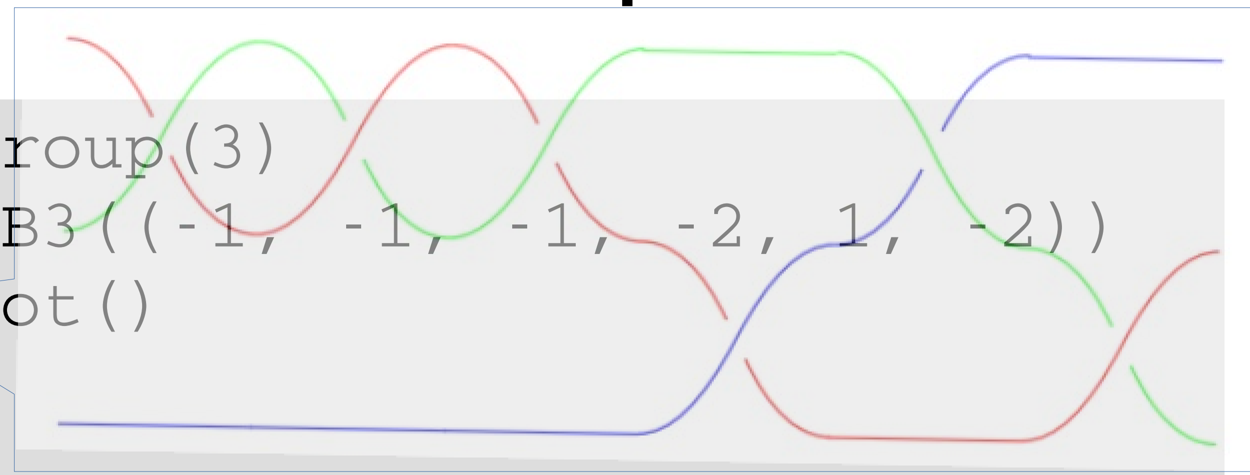

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sage: knot5_2 = Link(braid5_2)
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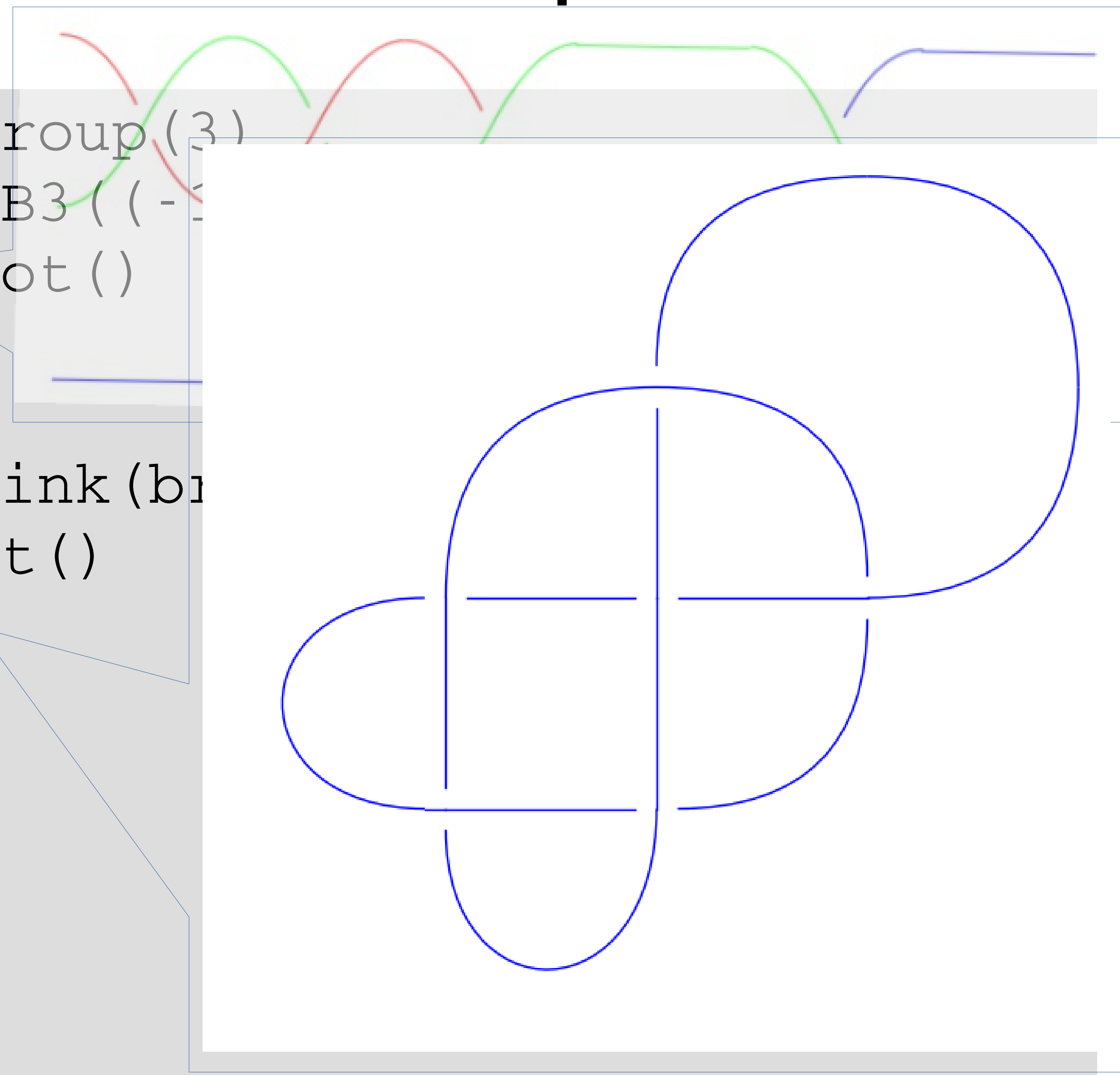

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Why are cubic braid groups interesting?

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The HOMFLY-PT Polynomial

The *HOMFLY-PT polynomial* $H(L)(a,z)$ (see [HOMFLY] and [PT]) of a knot or link L is defined by the skein relation

$$aH\left(\nwarrow\!\!\!\!\nearrow\right) - a^{-1}H\left(\nwarrow\!\!\!\!\nearrow\right) = zH\left()\,(\right)$$

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$$L(s_+) = aL(s), \qquad L(s_-) = a^{-1}L(s)$$

(here s is a strand and $s_\pm$ is the same strand with a $\pm$ kink added) and

$$L(\times) + L(\times) = z\left(L()\,() + L(\asymp)\right)$$

and by the initial condition $L(U) = 1$ where U is the unknot $\bigcirc$.

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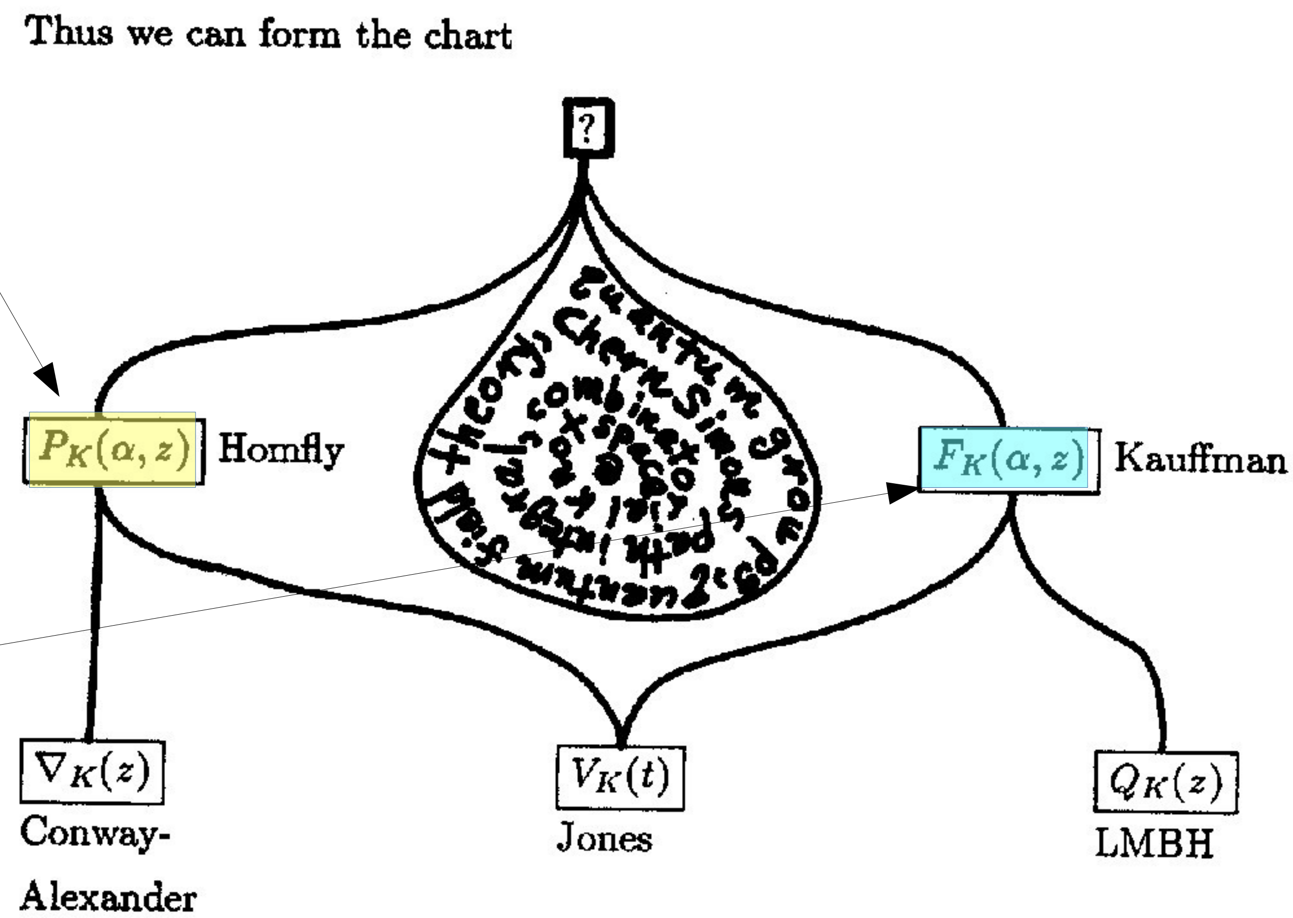

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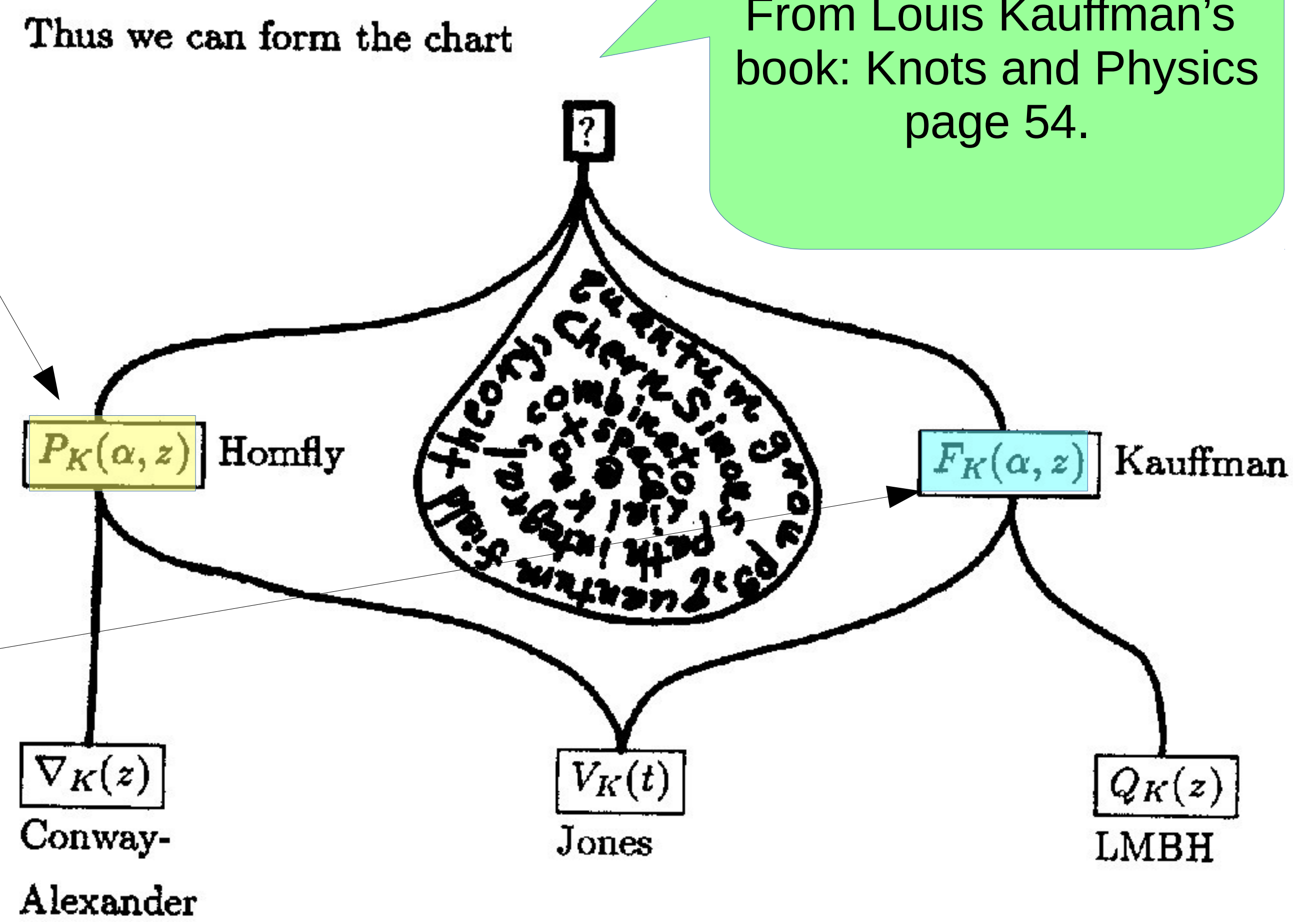

From Louis Kauffman's book: Knots and Physics page 54.

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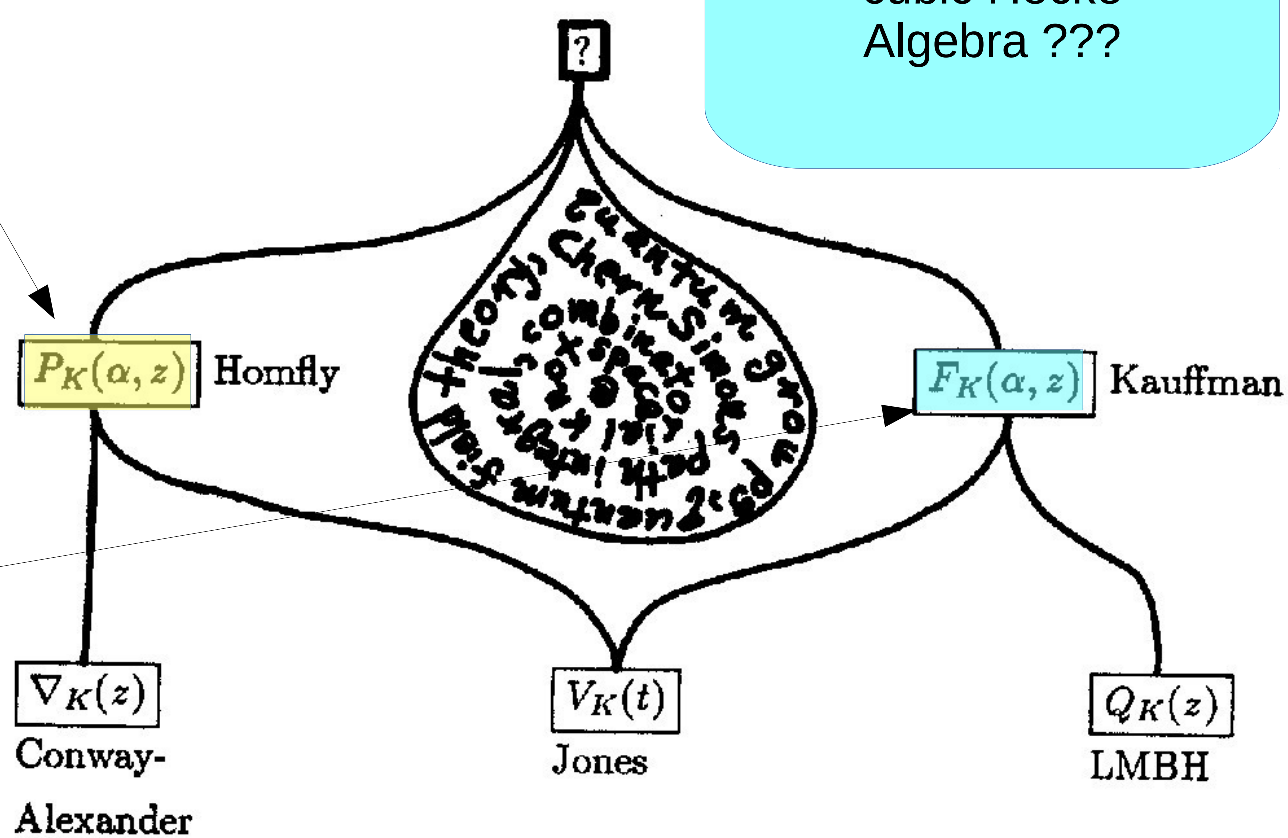
??? Markov trace on cubic Hecke Algebra ???

Thus we can form the chart



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In "Einige endliche Faktorgruppen der Zopfgruppen" (Math. Z., 163 (1978), 291-302) J. Assion considered two series $S(m)$ and $U(m)$ of finite dimensional factors of these groups. The additional relations on the braid group generators $\{s_1, \cdots, s_{m-1}\}$ are

$$\begin{array}{rlll} s_3 s_1 t_2 s_1 t_2^{-1} t_3 t_2 s_1 t_2^{-1} t_3^{-1} = 1 & \text{for} & m >= 5 & \text{in case of} \quad S(m) \\ t_1 t_3 = 1 & \text{for} & m >= 5 & \text{in case of} \quad U(m) \end{array}$$

where $t_i = (s_i s_{i+1})^3$. He showed that each series of finite cubic braid group factors must be an epimorhic image of one of his two series, as long as the groups with less than 5 strands are the full cubic braid groups, whereas the group on 5 strands is not. He realized the groups $S(m)$ as symplectic groups over $GF(3)$ (resp. subgroups therein) and $U(m)$ as general unitary groups over $GF(4)$ (resp. subgroups therein).

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Functionality of the class

This class implements all the groups conidered by Coxeter and Assion as finitely presented groups (via the gap interface) together with the classical realizations given by the authors. It also contains the coercion maps between the two ways of realization. In addition the user can construct other realizations and maps to matrix groups with help of the burau representation. In case gap3 and CHEVIE are installed under sage version 7.2 (or later) the reflection groups via the gap3 interface are availlable, too. The methods for all this functionality are:

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Cubic Braid group on 3 strands
sage: C3Cl = C3.as_classical_group(); C3Cl
Subgroup of Unitary Group of degree 2 over Universal
Cyclotomic Field with respect to hermitian form
[-E(12)^7 + E(12)^11                  -1]
[                 -1 -E(12)^7 + E(12)^11] generated by:
([    E(3) E(12)^11]
[       0        1], [       1        0]
[E(12)^11     E(3)])
braid5_2inC3 = C3(braid5_2)
sage: braid5_2inC3Cl = C3Cl(braid5_2inC3); braid5_2inC3Cl
[-E(3)^2 E(12)^7]
[      0      -1]
sage: braid5_2inC3back = C3(braid5_2inC3Cl); braid5_2inC3back
c0*c1*c0^2*c1
sage: braid5_2inC3 == braid5_2inC3back
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sage: C3 = CubicBraidGroup (3) ; C3
Cubic Braid group on 3 strands
sage: C3Cl = C3.as_classical_group(); C3Cl
Subgroup of Unitary Group of degree 2 over Universal
Cyclotomic Field with respect to hermitian form
[-E(12)^7 + E(12)^11                    -1]
[                  -1 -E(12)^7 + E(12)^11] generated by:
([    E(3) E(12)^11]
[       0         1], [       1         0]
[E(12)^11      E(3)])
braid5_2inC3 = C3(braid5_2)
sage: braid5_2inC3Cl = C3Cl(braid5_2inC3); braid5_2inC3Cl
[-E(3)^2 E(12)^7]
[      0      -1]
sage: braid5_2inC3back = C3(braid5_2inC3Cl); braid5_2inC3back
c0*c1*c0^2*c1
sage: braid5_2inC3 == braid5_2inC3back
True
sage: braid5_2inC3.braid() == braid5_2inC3back.braid()
False
```

Examples of general cubic braid groups

```
sage: from cubic_braid import *
sage: C3 = CubicBraidGroup (3) ; C3
Cubic Braid group on 3 strands
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[-E(3)^2 E(12)^7]
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sage: braid5_2inC3back = C3(braid5_2inC3Cl); braid5_2inC3back
c0*c1*c0^2*c1
sage: braid5_2inC3 == braid5_2inC3back
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```

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[-E(3)^2 E(12)^7]
[      0      -1]
sage: braid5_2inC3back = C3(braid5_2inC3Cl); braid5_2inC3back
c0*c1*c0^2*c1
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[-E(12)^7 + E(12)^11                    -1]
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([    E(3) E(12)^11]
[       0         1], [        1         0]
[E(12)^11      E(3)])
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[-E(3)^2 E(12)^7]
[      0      -1]
sage: braid5_2inC3back = C3(braid5_2inC3Cl); braid5_2inC3back
c0*c1*c0^2*c1
sage: braid5_2inC3 == braid5_2inC3back
True
sage: braid5_2inC3.braid() == braid5_2inC3back.braid()
False
```

converting braid5_2 to the classical realization

Examples of general cubic braid groups

```
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sage: C3 = CubicBraidGroup (3) ; C3
Cubic Braid group on 3 strands
sage: C3Cl = C3.as_classical_group(); C3Cl
Subgroup of Unitary Group of degree 2 over Universal
Cyclotomic Field with respect to hermitian form
[-E(12)^7 + E(12)^11                  -1]
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([     E(3) E(12)^11]
[        0         1], [        1         0]
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braid5_2inC3 = C3(braid5_2)
sage: braid5_2inC3Cl = C3Cl(braid5_2inC3); braid5_2inC3Cl
[-E(3)^2 E(12)^7]
[      0      -1]
sage: braid5_2inC3back = C3(braid5_2inC3Cl); braid5_2inC3back
c0*c1*c0^2*c1
sage: braid5_2inC3 == braid5_2inC3back
True
sage: braid5_2inC3.braid() == braid5_2inC3back.braid()
False
```

Converting back

Examples of general cubic braid groups

```
braid5_2inC3 = C3(braid5_2)
sage: braid5_2inC3Cl = C3Cl(braid5_2inC3); braid5_2inC3Cl
[-E(3)^2 E(12)^7]
[      0      -1]
sage: braid5_2inC3back = C3(braid5_2inC3Cl); braid5_2inC3back
c0*c1*c0^2*c1
sage: braid5_2inC3 == braid5_2inC3back
True
sage: braid5_2inC3.braid() == braid5_2inC3back.braid()
False
sage: braid5_2inC3back.braid().plot()
Launched png viewer for Graphics object consisting of 20
graphics primitives
sage: Link(braid5_2inC3back.braid()).plot()
Launched png viewer for Graphics object consisting of 24
graphics primitives
```

Converting back

Examples of general cubic braid groups

```
braid5_2inC3 = C3(braid5_2)
sage: braid5_2inC3Cl = C3Cl(braid5_2inC3); braid5_2inC3Cl
[-E(3)^2 E(12)^7]
[      0       -1]
sage: braid5_2inC3back = C3(braid5_2inC3Cl); braid5_2inC3back
c0*c1*c0^2*c1
sage: braid5_2inC3 == braid5_2inC3back
True
sage: braid5_2inC3.braid() == braid5_2inC3back.braid()
False
sage: braid5_2inC3back.braid().plot()
Launched png viewer for Graphics object consisting of 20
graphics primitives
sage: Link(braid5_2inC3back.braid()).plot()
Launched png viewer for Graphics object consisting of 24
graphics primitives
```

Examples of general cubic braid groups

```
braid5_2inC3 = C3(braid5_2)
sage: braid5_2inC3Cl = C3Cl(braid5_2inC3); braid5_2inC3Cl
[-E(3)^2 E(12)^7]
[      0      -1]
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sage: braid5_2inC3.braid() == braid5_2inC3back.braid()
False
sage: braid5_2inC3back.braid().plot()
Launched png viewer for Graphics object consisting of 20
graphics primitives
sage: Link(braid5_2inC3back.braid()).plot()
Launched png viewer for Graphics object consisting of 24
graphics primitives
```

pre-image in the braid group

Examples of general cubic braid groups

```
braid5_2inC3 = C3(braid5_2)
sage: braid5_2inC3Cl = C3Cl(braid5_2inC3); braid5_2inC3Cl
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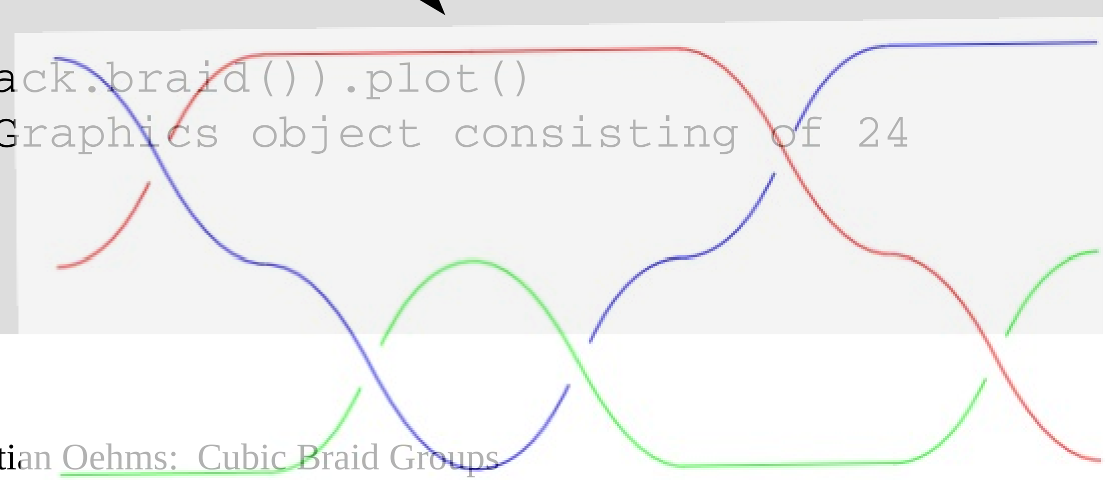
graphics primitives
sage: Link(braid5_2inC3back.braid()).plot()
Launched png viewer for Graphics object consisting of 24
graphics primitives
```



Examples of general cubic braid groups

```
braid5_2inC3 = C3(braid5_2)
sage: braid5_2inC3Cl = C3Cl(braid5_2inC3); braid5_2inC3Cl
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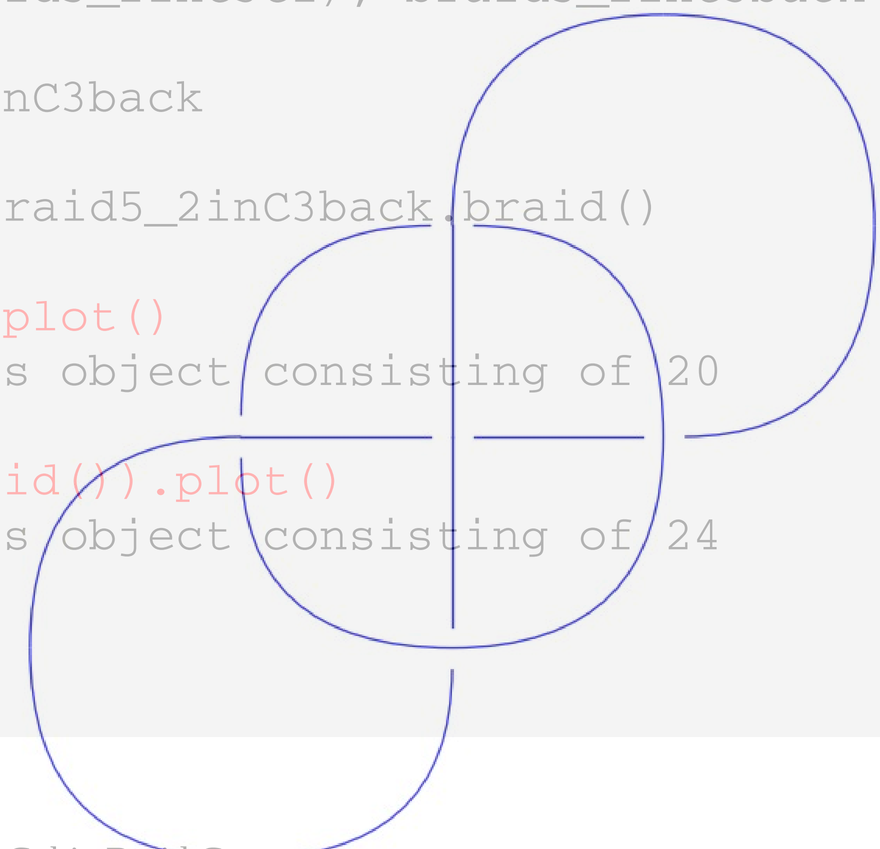
```



Examples of Assion Groups

```
sage: S3 = AssionGroupS(3); S3
Assion group on 3 strands of type S
sage: U3 = AssionGroupU(3); U3
Assion group on 3 strands of type U
sage: C3.is_isomorphic(S3)
True
sage: C3.is_isomorphic(U3)
True
sage: C3 == S3
False
sage: C3 == U3
False
sage: S3 == U3
False
```

Examples of Assion Groups

```
sage: S3 = AssionGroupS(3); S3
Assion group on 3 strands of type S
sage: U3 = AssionGroupU(3); U3
Assion group on 3 strands of type U
sage: C3.is_isomorphic(S3)
True
sage: C3.is_isomorphic(U3)
True
sage: C3 == S3
False
sage: C3 == U3
False
sage: S3 == U3
False
```

Examples of Assion Groups

```
sage: S3 = AssionGroupS(3); S3
Assion group on 3 strands of type S
sage: U3 = AssionGroupU(3); U3
Assion group on 3 strands of type U
sage: C3.is_isomorphic(S3)
True
sage: C3.is_isomorphic(U3)
True
sage: C3 == S3
False
sage: C3 == U3
False
sage: S3 == U3
False
```

Assion Groups of higher order

```
sage: S5.order()
51840
sage: U5.order()
77760
sage: C5.order()
155520
sage: S5Cl = S5.as_classical_group(); S5Cl
Symplectic Group of degree 4 over Finite Field
of size 3
sage:
sage: U5Cl = U5.as_classical_group(); U5Cl
General Unitary Group of degree 4 over Finite
Field in a of size 2^2
```

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```
sage: S5.order()
51840
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sage:
sage: U5Cl = U5.as_classical_group(); U5Cl
General Unitary Group of degree 4 over Finite
Field in a of size 2^2
```

Overview

Contents

Exceptions in Assion Group Series

```
sage: U3Cl = U3.as_classical_group(); U3Cl
Subgroup of (The projective general unitary
group of degree 3 over Finite Field of size 2)
generated by [(1,7,6)(3,19,14)(4,15,10)(5,11,18)
(12,16,20), (1,12,13)(2,15,19)(4,9,14)(5,18,8)
(6,21,16)]
sage: U3Clemb =
U3.as_classical_group(embedded=True); U3Clemb
Matrix group over Finite Field in a of size 2^2
with 2 generators (
[0 0 a]  [a + 1     a     a]
[0 1 0]  [    a a + 1     a]
[a 0 a], [    a     a a + 1]
)
```

Exceptions in Assion Group Series

```
sage: U3Cl = U3.as_classical_group(); U3Cl
Subgroup of (The projective general unitary
group of degree 3 over Finite Field of size 2)
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[0 0 a]   [a + 1      a      a]
[0 1 0]   [    a  a + 1      a]
[a 0 a],  [    a      a  a + 1]
)
```

Overview

Contents

Other Realizations

```
sage: C3RG = C3.as_reflection_group(); C3RG
Irreducible complex reflection group of rank 2 and
type ST4
sage: coxelem = C3RG.coxeter_element(); coxelem
(1,7,6,12,23,20)(2,8,17,24,9,5)(3,16,10,19,15,21)
(4,14,11,22,18,13)
sage: C3(coxelem)
c0*c1
sage: C3M5 = sage: C3M5 =
C3.as_matrix_group(characteristic=5); C3M5
Matrix group over Finite Field in rI of size 5^2
with 2 generators (
[2*rI + 2 3*rI + 4        0]  [       1        0        0]
[       1        0        0]  [       0 2*rI + 2 3*rI + 4]
[       0        0        1], [       0        1        0]
)
```

Other Realizations

```
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sage: C3(coxelem)
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sage: C3M5 = sage: C3M5 =
C3.as_matrix_group(characteristic=5); C3M5
Matrix group over Finite Field in rI of size 5^2
with 2 generators (
[2*rI + 2 3*rI + 4        0]  [       1        0        0]
[       1        0        0]  [       0 2*rI + 2 3*rI + 4]
[       0        0        1], [       0        1        0]
)
```

Other Realizations

```
sage: C3RG = C3.as_reflection_group(); C3RG
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(4,14,11,22,18,13)
sage: C3(coxelem)
c0*c1
sage: C3M5 = sage: C3M5 =
C3.as_matrix_group(characteristic=5); C3M5
Matrix group over Finite Field in rI of size 5^2
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[2*rI + 2 3*rI + 4        0]   [      1          0          0]
[       1        0        0]   [      0 2*rI + 2 3*rI + 4]
[       0        0        1],  [      0          1          0]
)
```

Other Realizations

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sage: C3RG = C3.as_reflection_group(); C3RG
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with 2 generators (
[2*rI + 2 3*rI + 4        0]   [      1        0        0]
[       1        0        0]   [      0 2*rI + 2 3*rI + 4]
[       0        0        1],  [      0        1        0]
)
```

Implementation

```
sage/CubicBraidGroup $ ls  -ltr lib/*.py
      0 Dez 30  2016 lib/__init__.py
  27620 Apr  3  2017 lib/utils_sys.py
  41473 Apr  3  2017 lib/utils_gap_interface.py
  17263 Apr  3  2017 lib/local_braid.py
  21185 Apr  3  2017 lib/local_matrix_group.py
  23680 Apr  6  2017 lib/local_permgroup.py
 133736 Jun 10 22:17 lib/cubic_braid.py
sage/CubicBraidGroup $
```

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sage/CubicBraidGroup $
```

These modules contain suggestions for improvements of sage

Implementation

```
sage/CubicBraidGroup $ ls  -ltr lib/*.py
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 133736 Jun 10 22:17 lib/cubic_braid.py
sage/CubicBraidGroup $
```

For example Tickets #25686, #25706

Sage-problems I had to solve

the following I noticed when I tried to obtain the natural projection from finite unitary and symplectic groups to the corresponding projective groups

Sage-problems I had to solve

```
sage: G = GU(3,2); G
General Unitary Group of degree 3 over Finite
Field in a of size 2^2
sage: MG = G.as_matrix_group(); MG
Matrix group over Finite Field in a of size 2^2
with 2 generators (
[a 0 0]   [a 1 1]
[0 1 0]   [1 1 0]
[0 0 a],  [1 0 0]
)
sage: mg = MG.an_element(); mg
[a + 1     a     a]
[    1     1     0]
[    a     0     0]
```

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sage: mg = MG.an_element(); mg
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)
sage: mg = MG.an_element(); mg
[a + 1     a     a]
[    1     1     0]
[    a     0     0]
```

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[a + 1     a     a]
[    1     1     0]
[    a     0     0]
```

Sage-problems I had to solve

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sage: PG(mg)
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Now, try to obtain
the image of mg
in the permutation group

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TypeError:
..................
'sage.groups.matrix_gps.group_element.MatrixGrou
pElement_gap' object is not iterable
```

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```
sage: conv_ori = PG.convert_map_from(MG);
conv_ori
Call morphism:
  From: Matrix group over Finite Field in a of
size 2^2 with 2 generators (….)
  To:   Permutation Group with generators
[(2,3,5)(4,7,12)…. ]
sage: img_mg = conv_ori(mg)
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Second try

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The problem consists of two issues:

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2) Make the conversion work with __call__

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sage: MGgens = MG.gap().GeneratorsOfGroup()
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sage: conv_map_gap = gap_hom(MGgens, PGgens)
sage: def conv_map_func(elem):
....:     res = conv_map_gap.ImageElm(elem.gap())
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sage: conv_map = Hom(MG, PG)(conv_map_func)
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Try to construct the homomorphism via GAP

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(1,2,6,19,35,33)(3,9,26,14,31,23)(4,13,5)(7,22,17)
(8,24,12)(10,16,32,27,20,28)(11,30,18)
(15,25,36,34,29,21)
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```

Idea: Integrate this in the as_permutation_group method of FinitelyGeneratedMatrixGroup_gap

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What would be the right way to solve this problem in Sage?