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A NOTE ON HAMILTONIAN CIRCUITS*

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The purpose of this note is to prove the following

Theorem 1. *Let G be a graph with at least three vertices. If, for some s, G is s-connected and contains no independent set of more than s vertices, then G has a Hamiltonian circuit.*

This theorem is sharp as the complete bipartite graph $K(s, s+1)$ is s-connected, contains no independent set of more than $s+1$ vertices and has no Hamiltonian circuit. Similarly, the Petersen graph is 3-connected, contains no independent set of more than four vertices and has no Hamiltonian circuit.

Proof. Let G satisfy the hypothesis of Theorem 1. Clearly, G contains a circuit; let C be the longest one. If G has no Hamiltonian circuit, there is a vertex x with $x \notin C$. Since G is s-connected, there are s paths starting at x and terminating in C which are pairwise disjoint apart from x and share with C just their terminal vertices $x_1, x_2, ..., x_s$ (see [1], Theorem 1). For each $i = 1, 2, ..., s$, let y_i be the successor of x_i in a

* This note was written in Professor Richard K. Guy's car on the way from Pullman to Spokane, Wash. The autors wish to express their gratitude to Mrs. Guy for smooth driving.

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fixed cyclic ordering of C. No y_i is adjacent to x - otherwise we would replace the edge x_iy_i in C by the path going from x_i to y_i outside C (via x) and obtain a longer circuit. However, G contains no independent set of $s+1$ vertices and so there is an edge y_iy_j. Delete the edges x_iy_i, x_jy_j from C and add the edge y_iy_j together with the path going from x_i to x_j outside C. In this way we obtain a circuit longer than C, which is a contradiction.

For s relatively large with respect to the number of vertices of G, our Theorem 1 follows from a stronger statement due to Nash-Williams and Bondy ([2], Lemma 4):

Let G be a graph with n vertices, $n \geqslant 3$. Let G contain no vertex of degree smaller than k where k is an integer such that $k \geqslant \frac{1}{3}(n+2)$. Then G either has a Hamiltonian circuit, or is separable, or has $k+1$ independent vertices.

As an easy consequence of Theorem 1 we obtain

Theorem 2. *Let G be an s-connected graph with no independent set of $s+2$ vertices. Then G has a Hamiltonian path.*

Proof. Indeed, if G satisfies the hypothesis of Theorem 2, then $G+x$ (the graph obtained from G by adding a new vertex x and joining it to all the vertices of G) satisfies the hypothesis of Theorem 1 with $s+1$ in place of s. Therefore $G+x$ has a Hamiltonian circuit and G has a Hamiltonian path. The complete bipartite graph $K(s, s+2)$ shows that Theorem 2 is sharp.

The technique used in the proof of Theorem 1 yields also

Theorem 3. *Let G be an s-connected graph containing no independent set of s vertices. Then G is Hamiltonian-connected (i.e. every pair of vertices is joined by a Hamiltonian path).*

Proof. Let there be a counterexample G. Then G contains three vertices x, y, z such that $x \notin P$ for a longest path P joining y to z. Again, we find s paths from x to P, their terminal vertices being $x_1, \ldots, x_s$. We may assume $x_i \neq z$ for $i < s$ and denote the successor (in the direction from y to z) of each x_i ($i < s$) by y_i. Since G has no s independent vertices, there is an edge xy_i or y_iy_j. In both cases we find a path joining y to z and longer than P which is a contradiction. The graph $K(s, s)$ shows that Theorem 3 is sharp.

References

[1] G.A. Dirac, Généralisation du théorème de Menger, C.R. Acad. Sci. Paris 250 (26) (1960) 4252–4253.

[2] J.A. Nash-Williams, Edge-disjoint Hamiltonian circuits in graphs with vertices of large valency, in: L. Mirsky, ed., Studies in pure mathematics (papers presented to Richard Rado) (Academic Press, New York, 1971).