

# AN ATLAS OF GRAPHS

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*Italic entries refer to the pictures of graphs*

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## Index of abbreviations

We list here the abbreviations for the various types of graphs depicted in this *Atlas*.

A	acyclic digraphs	298
B	connected bipartite graphs	191
Bc	connected bicubic graphs	157
Bn	binary trees	101
C	connected cubic graphs	127
Cp	convex cubic polyhedral graphs	159
Ct	connected cubic transitive graphs	161
D	digraphs	292
E	Eulerian graphs	197
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F	connected quintic graphs	154
G	graphs	8
H	homeomorphically irreducible trees	84
Id	identity trees	97
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# Tables of graph numbers

*Graphs with  $n$  vertices, for  $n = 1, 2, \dots, 25$*

$n$	graphs
1	1
2	2
3	4
4	11
5	34
6	156
7	1044
8	12346
9	2 74668
10	120 05168
11	10189 97864
12	16 50911 72592
13	5050 20313 67952
14	29 05415 56572 35488
15	31426 48596 98043 08768
16	640 01015 70452 75578 94928
17	24 59358 64153 53293 26837 19776
18	1 78757 77251 45611 70054 78781 90848
19	24637 80925 31250 04524 38300 74914 32768
20	6454 90122 79579 98418 56164 63849 07427 49440
21	3222 02728 99808 98343 35022 44253 75528 36160 97664
22	3070 84648 30941 44300 63756 85171 87105 41058 66578 14272
23	5599 24939 69979 20805 97976 38081 94621 79812 27634 84589 81632
24	19570 49063 02078 44792 21748 62416 72625 60041 22075 26706 33657 54368
25	1 31331 39356 98955 19432 16154 84058 16890 14638 92147 06146 48338 04585 76384

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## 4 Graphs

### *Connected graphs with $n$ vertices, for $n = 1, 2, \dots, 25$*

<i>n</i>	<i>connected graphs</i>													
1										1				
2										1				
3										2				
4										6				
5										21				
6										112				
7										853				
8										11117				
9									2	61080				
10									117	16571				
11									10067	00565				
12									16	40598 30476				
13									5033	59078 69219				
14									29	00348 74628 48061				
15									31397	38114 27612 41960				
16									639	69560 11322 51761 76277				
17									24	58718 31682 08402 65195 28568				
18									1	78733 17252 48899 08889 02005 76580				
19									24636	02142 93998 67655 32265 07596 81644				
20									6454	65483 19872 27994 26731 12879 45022 83004				
21									3221	96273 85046 58981 82320 44119 08215 73234 36797				
22									3070	81426 21757 29723 89556 84543 99186 82950 95501 71755				
23									5599	43868 82108 80679 65169 46729 81026 45124 90467 68525 69700				
24									19570	43463 52067 86942 41449 39394 11275 95390 46378 87524 67290 14803				
25	1	31331	19786	44292	67343	03846	15711	21231	90135	75463	49778	70337	47441	10480

*2-connected graphs with  $n$  vertices, for  $n = 3, 4, \dots, 20$*

*raphs*  
 1  
 1  
 2  
 6  
 21  
  
 112  
 853  
 11117  
 61080  
 16571  
  
 00565  
 30476  
 69219  
 48061  
 41960  
  
 76277  
 28568  
 76580  
 81644  
 83004  
  
 36797  
 71755  
 69700  
 14803  
 10480

$n$	2-connected graphs
3	1
4	3
5	10
6	56
7	468
8	7123
9	1 94066
10	97 43542
11	9009 69091
12	15 36203 33545
13	4843 29391 50704
14	28 36182 44883 94169
15	30995 89080 60333 80784
16	635 01635 42910 95975 04951
17	24 48520 79292 07337 60104 11280
18	1 78316 05940 69429 92595 28247 34641
19	24603 88705 13509 45867 49281 66639 58981
20	6449 97704 30459 87615 31891 39098 95833 04810

*3-connected graphs with  $n$  vertices, for  $n = 4, 5, \dots, 20$*

$n$	3-connected graphs
4	1
5	3
6	17
7	136
8	2388
9	80890
10	51 14079
11	5732 73505
12	11 30951 67034
13	3958 25505 75765
14	24 90844 57930 58442
15	28560 40514 34958 19079
16	603 64410 13017 72230 14724
17	23 74039 33018 79995 83095 30349
18	1 75032 31373 55778 19015 80820 29500
19	24333 35881 36993 71350 71522 11074 64003
20	6408 11613 27875 27544 85012 44396 35795 01421





