All work on this lab should be the collective effort of all group members. Technology allowed on this lab includes: Desmos (`https://www.desmos.com/calculator`) and an approved TI calculator. This lab has 6 questions for a total of 57 points.

1. Write a delta-epsilon proof for each limit.

   (a) (5 points) $\lim_{x\to 3}(2x-3) = 3$

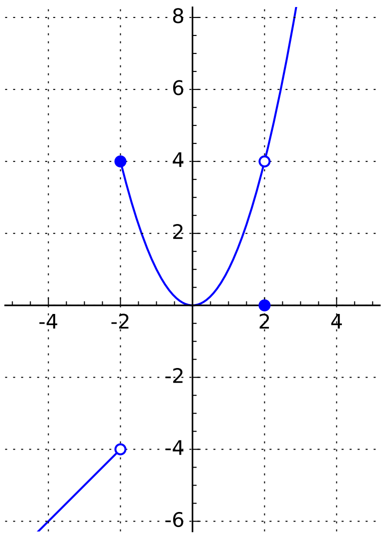
   (b) (5 points) $\lim_{x\to 2}\dfrac{x^2-4}{x-2} = 4$

2. (10 points) Consider the following piecewise-defined function.

$$f(x) = \begin{cases} 2x^2 + b, & \text{if } x \geq 1 \\ -x^3, & \text{if } x < 1 \end{cases}$$

Find the value of b such that $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$.

3. Use the graph below to answer questions about its continuity.



(a) (4 points) What type of continuity does the graph have at $x = -2$? Explain why.

Right / Left / Both / Neither

(b) (4 points) What type of continuity does the graph have at $x = 2$? Explain why.

Right / Left / Both / Neither

(c) (2 points) What type of discontinuity does the graph have at $x = -2$?

Jump / Infinite / Removable / None of these

(d) (2 points) What type of discontinuity does the graph have at $x = 2$?

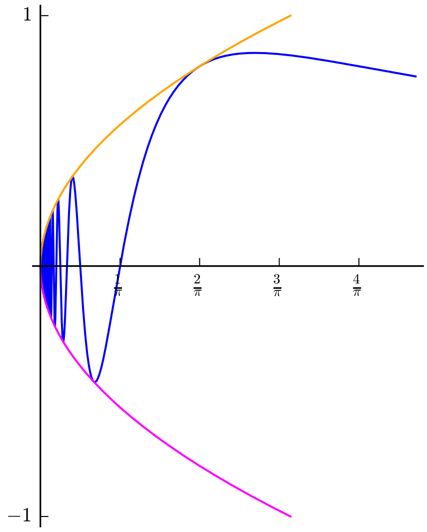
Jump / Infinite / Removable / None of these

4. (5 points) The **extreme value theorem** states the following:

> *"If a real-valued function f is continuous in the closed and bounded interval $[a, b]$, then f must attain a maximum and a minimum, each at least once."*

Does the function $f(x) = x^2 - 2x + 1$ satisfy the conditions of the extreme value theorem? Explain why or why not. [Hint: For this, you do **not** have to worry about left-hand and right-hand continuity.]

5. (points) [Looking Ahead] Consider the function $g(x) = \sqrt{x}\sin\left(\frac{1}{x}\right)$, who's graph is shown below.



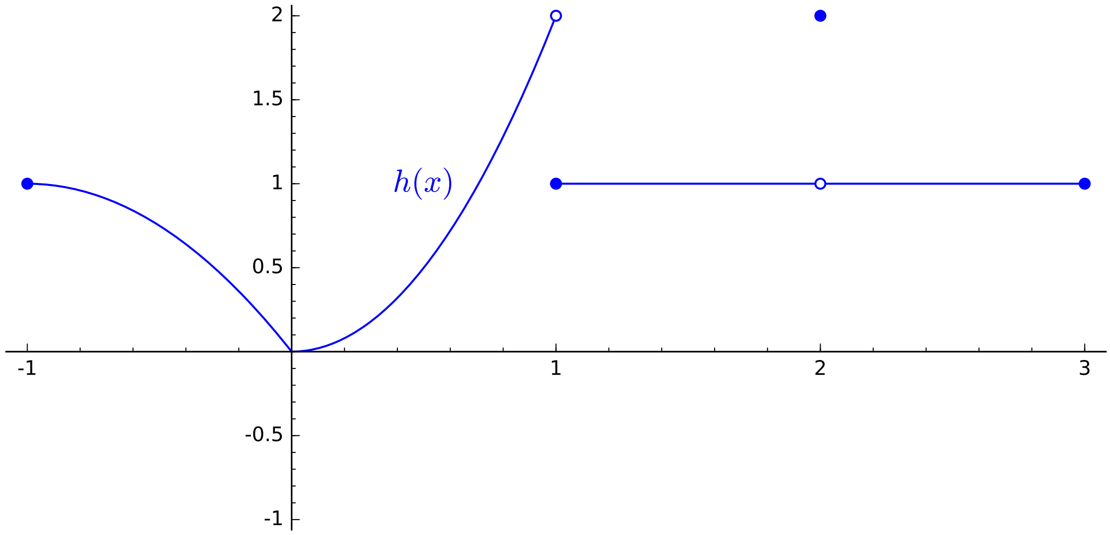
(a) (2 points) What do you expect $\lim_{x\to 0^+} g(x)$ to be?

(a) ____________

(b) (4 points) Why does $\lim_{x\to 0^-} g(x)$ not exist?

(c) (4 points) Use the domain of the function $g(x)$ to explain why this function is continuous on $(0, \infty)$, but not on $[0, \infty)$

6. (10 points) Answer TRUE / FALSE for each of following questions regarding the graph of the function $h(x)$.



(a) $\lim_{x \to -1^-} h(x) = 1$ TRUE / FALSE

(b) $\lim_{x \to 2} h(x) = 2$ TRUE / FALSE

(c) $\lim_{x \to 1^+} h(x) = 1$ TRUE / FALSE

(d) $\lim_{x \to 1^-} h(x) = 2$ TRUE / FALSE

(e) $\lim_{x \to -1^-} h(x) = 0$ TRUE / FALSE

(f) $\lim_{x \to c} h(x)$ exists at every c in the open interval $(-1, 1)$. TRUE / FALSE

(g) $\lim_{x \to c} h(x)$ exists at every c in the open interval $(1, 3)$. TRUE / FALSE

(h) $\lim_{x \to 3^+} h(x)$ does not exist. TRUE / FALSE

(i) $\lim_{x \to 1} h(x)$ does not exist. TRUE / FALSE

(j) The domain of $h(x)$ is $[-1, 3]$. TRUE / FALSE