TI-84 Skills for AP Calculus

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April 10, 2022

1 Introduction

The College Board AP Calculus tests demand a high level of calculator skill. In addition to evaluating derivatives and integrals, it is essential to know how to use the calculator to find zeros, intersections, and extremum. This tutorial will cover the basic skills, but also many shortcuts to save keystrokes and to preserve and enhance accuracy.

Absolute Essentials TL;DR

Things you must know how to do on the TI-84 for the AP Calc exam:

- Solve an equation with either the solver or the graphing utility
- Find the intersection(s) of two functions
- Find the derivative of a function at a point, and graph a derivative
- Evaluate a definite integral
- BC only: Graph a polar equation. Use your calculus and calculator skills to find a polar area.
- BC only: Find an arc length, with either regular functions or of a curve defined by parametric equations.
- Optional but fun: Evaluate a sequence or a series

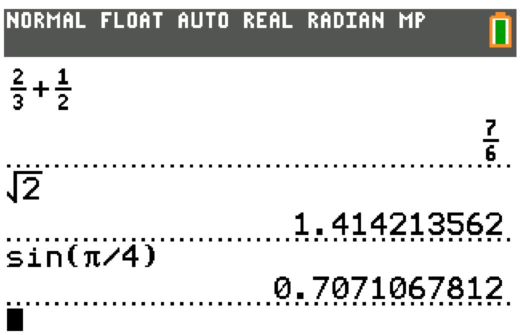
Getting Started

I recommend the official TI documentation. In our current age, where instruction manuals are usually an afterthought, the TI-84 user manuals are a pleasant surprise. They are well written, useful, and complete. You can find them at education.ti.com or download them here.

I'm using a TI-84 Plus CE for the screen shots in this tutorial, but most of the functions work on any TI-84. I am also a fan of the HP Prime, also allowed for the AP Calculus test. If you have another calculator, check the College Board's list of approved calculators, and investigate the necessary functions on your own device.

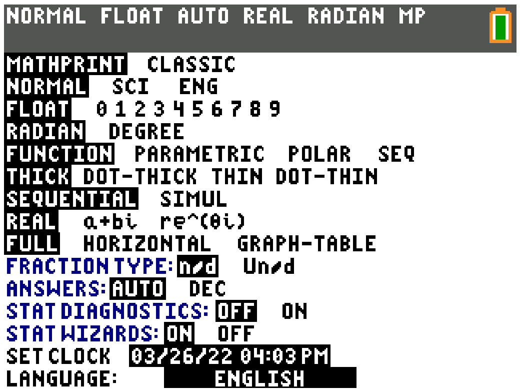
Turn your calculator on by pressing $\boxed{\text{on}}$.

The home screen should appear:



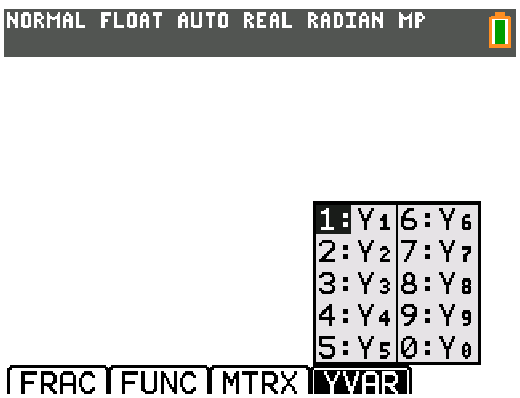
You can always return to the home screen by hitting the 'quit' key, i.e. by selecting the keys $\boxed{\text{2nd}}$ $\boxed{\text{mode}}$.

The settings I'm using for this tutorial are pictured here.



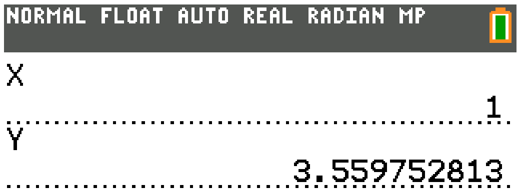
Fundamentals and Useful Tips

To open the shortcut menu press $\boxed{\text{alpha}}$ [F1]-[F4]. Use the left-right arrows and the number keys to navigate to different commands. This is the best way to select commonly used functions, or to reuse equations that have been previously entered in the graphing utility, such as Y_1, Y_2, etc.



Values previously calculated on the home screen can be retrieved by using the up arrow keys and $\boxed{\text{enter}}$. Even if you have cleared the screen, you should be able to continue to scroll up to see and retrieve a previous calculation. You should never need to retype a value or an equation, just use the built-in copy and paste.

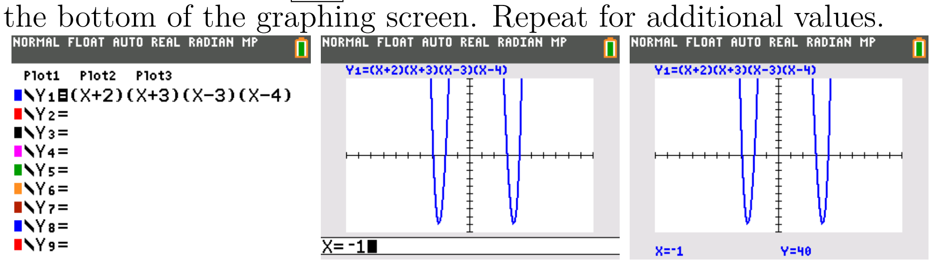
The x and y values of the current cursor position in the graphing utility can be accessed by typing X $\boxed{\text{enter}}$, and Y $\boxed{\text{enter}}$ on the home screen.



Calculators excel at saving time on repetitive calculations. Say you'd like to evaluate f(-1), f(0), f(1), and f(2) where $f(x) = (x+2)(x+3)(x-3)(x-4)$, then entering the function once into the graphing utility and using the calculator to evaluate at multiple points will save time.

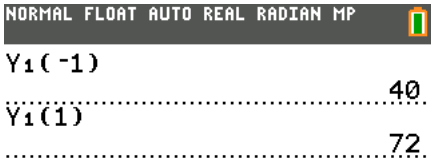
Enter the equation into the graphing utility, and graph it.

Press $\boxed{\text{trace}}$. Press $\boxed{\text{(-)}}$ $\boxed{1}$, and $\boxed{\text{enter}}$ to evaluate f(-1). Read the value at the bottom of the graphing screen. Repeat for additional values.



Alternately, after entering the equation in the graphing utility, open the "table" page, $\boxed{\text{2nd}}$ $\boxed{\text{graph}}$, to see the values of Y_1 with different x values. The "tblset" menu allows you to set various starting points, increments, and entry methods.

Finally, you can enter '$Y_1(-1)$' on the home screen command line, i.e. keys $\boxed{\text{alpha}}$ $\boxed{\text{trace}}$ $\boxed{1}$ $\boxed{(}$ $\boxed{\text{(-)}}$ $\boxed{1}$ $\boxed{)}$. To repeat you can copy the command from the previous line and replace the x value.



Example A person is standing on a hill overlooking a valley. Their eye level is five feet above the top of the hill. The shape of the valley below is given by $f(x) = 50\cos\left(\frac{x}{100}\right)$. The person's line of sight is tangent to the hill at point $A = \left(a, 50\cos\left(\frac{a}{100}\right)\right)$.

Find the equation of the tangent line, the location of point A, and determine whether the person can see the top of a flagpole, 25 feet tall, located at the bottom of the valley.

This is a great problem demonstrating many of the fundamental calculator skills needed. The calculus part of the problem is mostly done analytically with paper and pencil, while the calculator parts require the ability to evaluate and solve complex functions.

We know the function and the point on the function, so we can differentiate and use the point slope form to find the equation of the tangent line.

$$f(x) = 50\cos\left(\frac{x}{100}\right)$$

$$f'(x) = -\frac{1}{2}\sin\left(\frac{x}{100}\right)$$

$$y - 50\cos\left(\frac{a}{100}\right) = -\frac{1}{2}\sin\left(\frac{a}{100}\right)(x-a)$$

Since the tangent line also goes through the point $(0, 55)$, the person's eyes, we can also express the slope as follows:

$$m = \frac{55 - 50\cos\left(\frac{a}{100}\right)}{0-a}$$

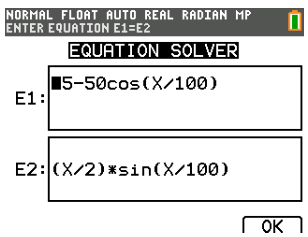
Setting the two expression for slope equal, we get

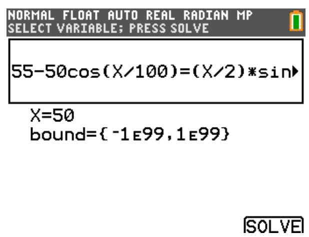
$$\frac{55 - 50\cos\left(\frac{a}{100}\right)}{0 - a} = -\frac{1}{2}\sin\left(\frac{x}{100}\right)$$

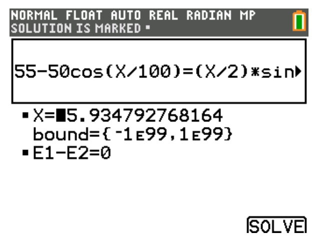
$$55 - 50\cos\left(\frac{a}{100}\right) = \frac{a}{2}\sin\left(\frac{a}{100}\right)$$

This is a transcendental equation, with no algebraic solution, so we need the calculator to solve for a.

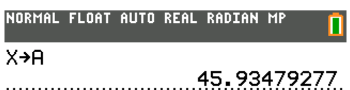
Start the 'numeric solver' app by typing $\boxed{\text{math}}$ $\boxed{\text{alpha}}$ $\boxed{\text{C}}$. Enter the left and right hand sides of the previous equation as shown, enter 50 as an initial guess, and a final $\boxed{\text{enter}}$ to solve.



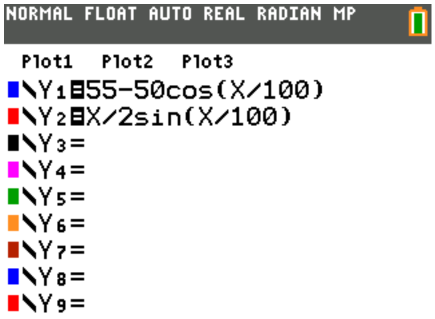


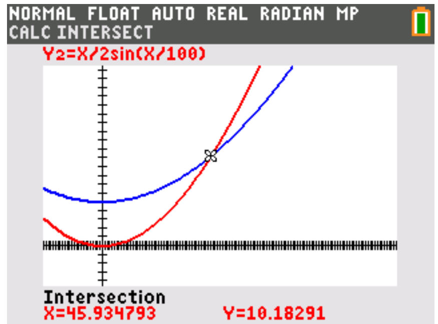


So $a = 45.934$. Save it to memory location A.

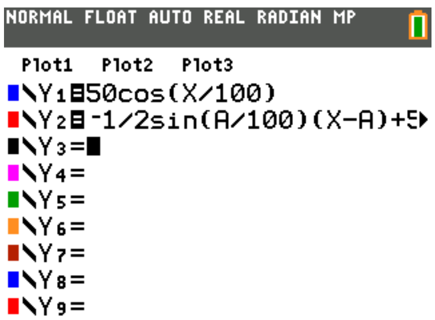


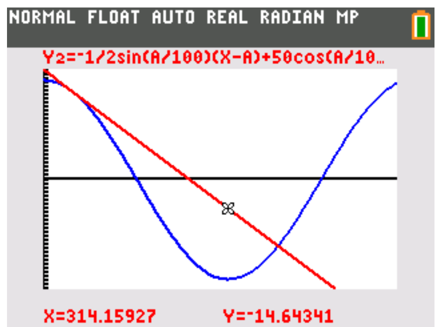
The other way to find a is with the graphing utility. Enter the the left and right side of the above equation as Y_1 and Y_2, then from the graph display start the 'calc' menu, $\boxed{\text{2nd}}$ $\boxed{\text{trace}}$, $\boxed{\text{5}}$ for 'intersect', answer the prompts and you'll see that $a = 45.934$.





Finally, to determine whether we can see the flagpole, we'll enter the equation of the original function and the new equation of the tangent line, with our calculated value for a. (Note the use of the saved value A.)





The tangent line value of y at $x = 100\pi$, the curve's minimum, is $y = -14.64$. The top of a 25 foot flagpole would be at $y = -25$, so we can not see the flag.

After entering the equations we could also have returned to the home screen and entered $Y_1(100\pi)$ $\boxed{\text{enter}}$ to directly evaluate the tangent line at the minimum.

2 Derivatives

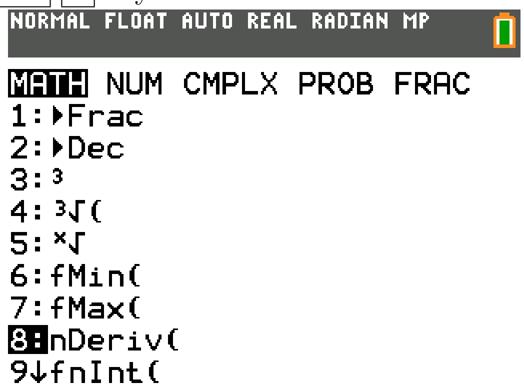
Here are some typical problems that involve values of derivatives. Follow along to see all the ways you can use your calculator to assist in solving them.

Example A particle moves along the x axis with velocity as a function of time given by $v(t) = e^t + \sin\left(\sqrt{t}\right)$. What is the acceleration at t=0.5?

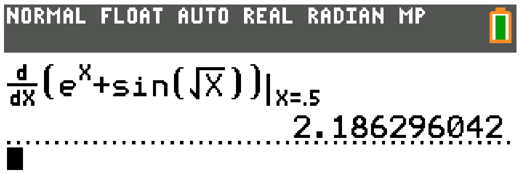
Acceleration is the derivative of velocity. Although we could approach the problem analytically, here we'll use the calculator to numerically evaluate the slope of the function at a point.

<u>Method 1.</u> Direct entry on the home screen.

From the home screen, use the 'nDeriv(' function, activated with the $\boxed{\text{math}}$ $\boxed{8}$ keys.

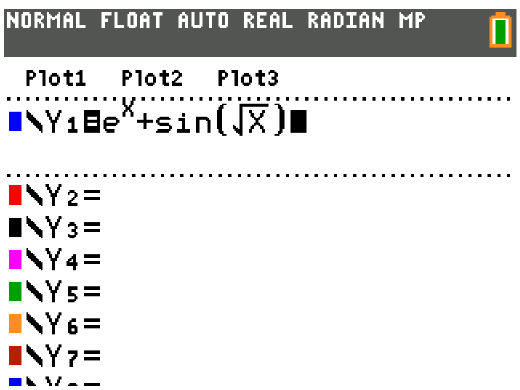


Fill in the blanks on the derivative template and hit $\boxed{\text{enter}}$. It should look like this.

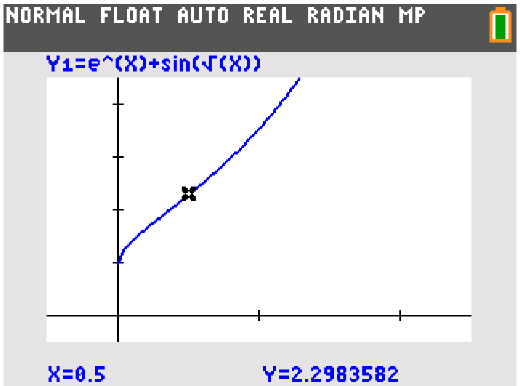


Method 2. Using the graphing utility. An advantage to graphing the function is that you can see whether the point of interest is at or near a discontinuity. Since the TI-84 is a numerical device, it can't correctly identify a discontinuity.

Use $\boxed{\text{y=}}$ to start the graphing utility. Enter the equation.

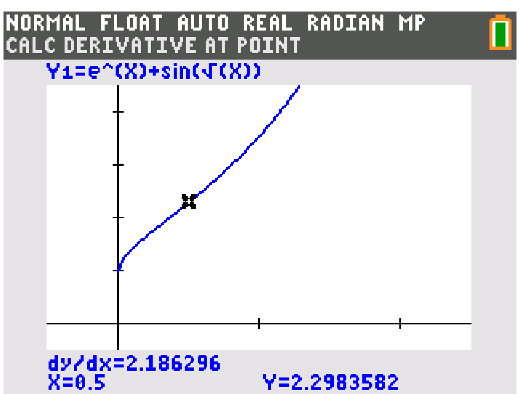


Press $\boxed{\text{graph}}$ to graph the function. Press $\boxed{\text{trace}}$ and the left arrow to move along the curve. Enter the number 0.5 directly to set the x value.



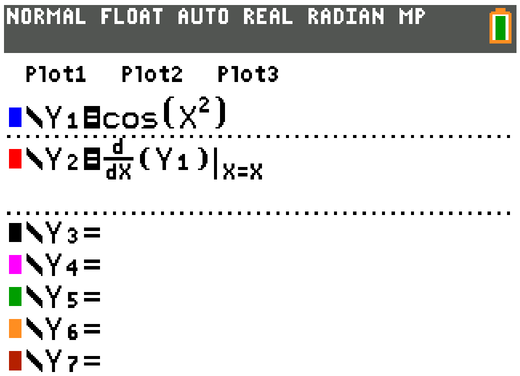
Press 'calc', i.e. $\boxed{\text{2nd}}$ $\boxed{\text{trace}}$ $\boxed{6}$, to find the derivative at the point.

Since we're already at the point x=0.5, hit $\boxed{\text{enter}}$ once more to display the derivative.

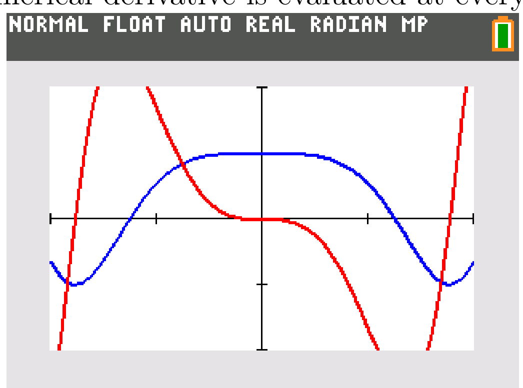


<u>Example</u> How many local extrema does the function $f(x) = \cos x^2$ have on the interval $x \in [-2, 2]$? How many points of inflection does it have?

Sometimes it is helpful to graph a derivative.



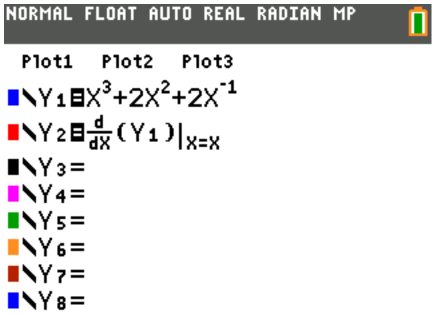
Note that in Y_2 the final entry in the derivative template is 'x', i.e the numerical derivative is evaluated at every value of x.

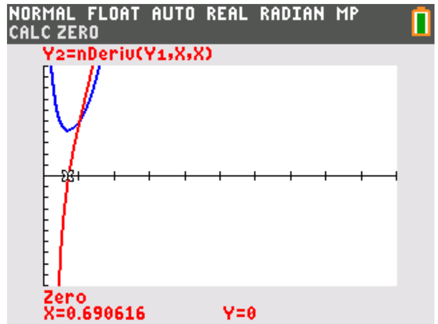


The blue curve is the original function and clearly it has three extrema. The red curve is the derivative, and perhaps it is easier to see that the derivative has three zeros, and changes sign at each. You can also see that the derivative has two extrema, hence its slope changes sign, so the function has two inflection points. We could graph a 2nd derivative using the same technique, but now enter $\frac{d}{dx}(Y_2)|_{x=x}$

Example If the velocity of a particle moving on the x-axis as a function of time is given by $v(t) = t^3 + 2t^2 + 2t^{-1}$ for $t \geq 0$, on what time intervals is the particle speeding up?

Again, graphing the function and the derivative will give a nice picture of the various intervals of speeding up / slowing down.

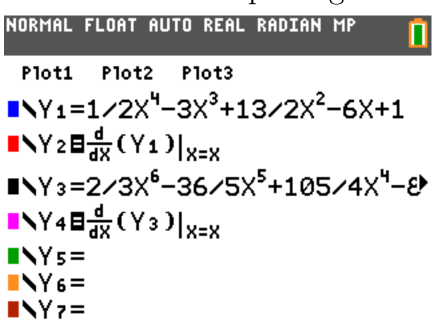


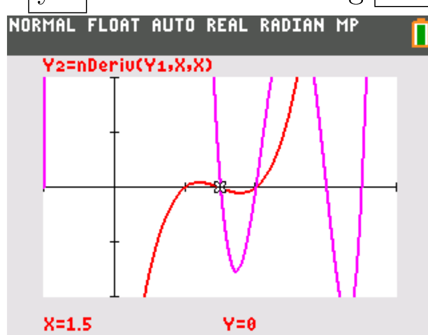


The velocity (blue curve) is always positive on the interval $x \in [0, 10]$. The acceleration (red) is positive for all x values greater than the zero at $x = 0.691$. Since velocity and acceleration have the same sign, the particle is speeding up on $x \in (0.691, 10]$.

Example If the position of a particle is given by the parametric equations $x(t) = \frac{1}{2}t^4 - 3t^3 + \frac{13}{2}t^2 - 6t + 1$ and $y(t) = \frac{2}{3}t^6 - \frac{36}{5}t^5 + \frac{105}{4}t^4 - \frac{80}{3}t^3 - \frac{99}{2}t^2 + 126t - 1$, how many times is the particle at rest on the interval $t \in [0, 3]$?

This problem seems hard at first because it is stated in parametric form, but all we need to do is take the derivative of both x- and y- positions to find v_x and v_y, then check whether they have any common zeros. If we graph both functions and both derivatives the plot will be busy, and the curves hard to distinguish, so we'll disable plotting the original functions by placing the cursor over the equal sign in the $\boxed{\text{y=}}$ window and hitting $\boxed{\text{enter}}$.





The particle is at rest when $t = 2$ and $t = \frac{3}{2}$.

3 Integrals

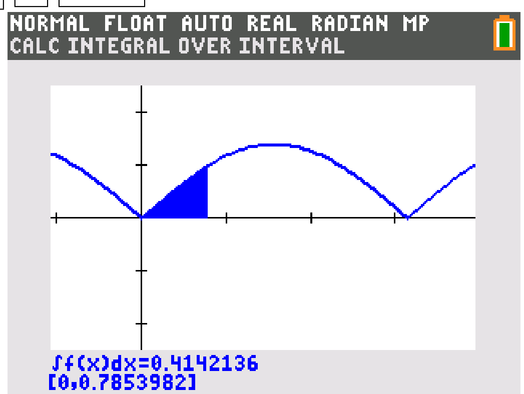
There are many types of questions on the AP Calculus test that involve finding the numerical values of derivatives. Here are a few examples.

Example Evaluate

$$\int_0^{\frac{\pi}{4}} \sqrt{1-\cos 2x}\, dx$$

Method 1. Use the graphing utility. Enter the equation $\sqrt{1-\cos 2x}$ into Y_1, and graph it.

Use the 'calc f4' key, and select $\boxed{7}$ for $\int f(x)dx$. The calculator will request 'Lower Limt?' Type $\boxed{0}$ $\boxed{\text{enter}}$. For 'Upper Limit?' type $\boxed{\text{2nd}}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{4}$ $\boxed{\text{enter}}$.

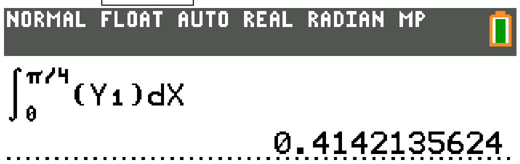


The value of the integral is displayed.

Method 2. Direct entry on the home screen. We could enter the equation directly as above, but in this case the equation is already saved in the calculator as Y_1.

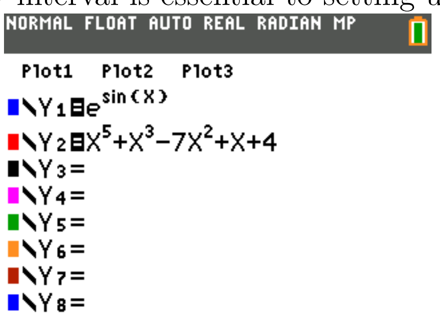
Press $\boxed{\text{math}}$ $\boxed{9}$ for the 'fnInt(' command, and the home screen should show

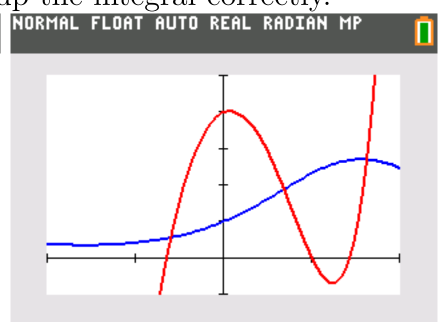
Fill in the blanks and $\boxed{\text{enter}}$. The best way to enter the function name Y_1 is with $\boxed{\text{alpha}}$ $\boxed{\text{trace}}$ $\boxed{1}$. Then $\boxed{\text{enter}}$ to evaluate.



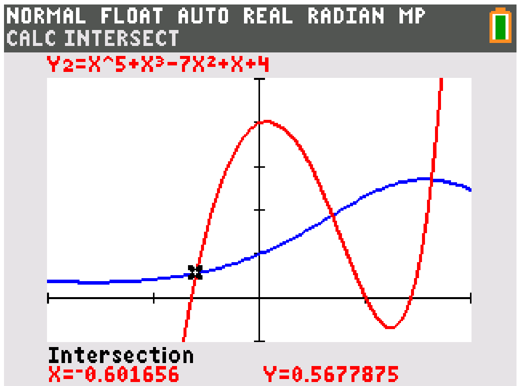
<u>Example</u> What is the area enclosed by the functions $y = e^{\sin x}$ and $y = x^5 + x^3 - 7x^2 + x + 4$?

Always graph the functions first; knowing which function is greatest on any interval is essential to setting up the integral correctly.

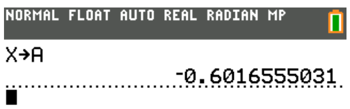




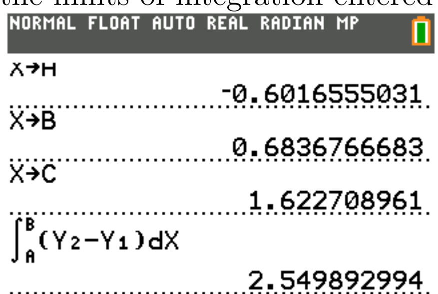
To find the intersections of the various regions, press 'calc', $\boxed{\text{2nd}}$ $\boxed{\text{trace}}$, 'intersect', $\boxed{5}$. Choose the first curve and the second curve, then when prompted for 'guess', move the cursor near the intersection you want, and $\boxed{\text{enter}}$.

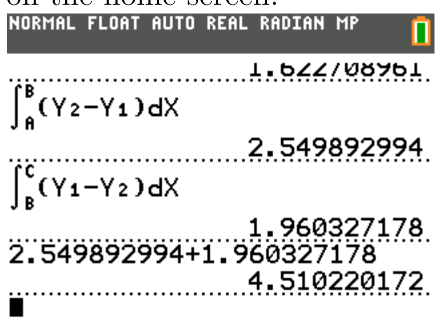


We'll save this leftmost intersection in the memory location A.



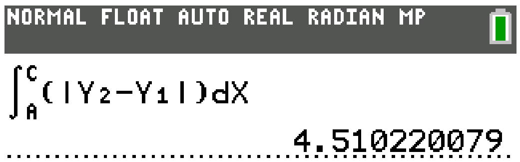
Repeat this procedure for the middle and rightmost intersections, saving them as B and C respectively. When this is complete, we'll use these values as the limits of integration entered on the home screen.





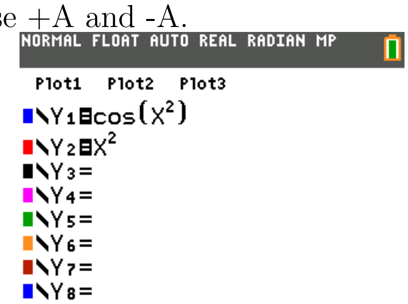
Note the order of the functions in the two integrals, and compare to the original graph. Finally, we add the results of the two numerical integrations. Saving values, and reusing values, is significantly quicker than retyping. No scratch paper is needed, and because accuracy is maintained, mistakes are minimized and the final result has maximum precision.

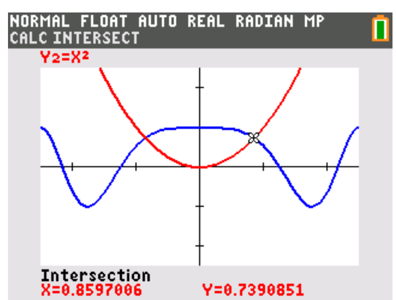
We could also use the absolute value of the difference of the functions, as follows.



Example Consider the region bounded by $f(x) = \cos(x^2)$ and $g(x) = x^2$ as the base of a solid. With cross sections perpendicular to the x-axis, and shaped like triangles with the height equal to the base, what is the volume?

Graphing the base always helps to imagine the problem, to plan a way forward, and therefore to evaluate the volume. The unusual aspect of this region, and perhaps this is a trick, is that it extends from the left side of the y-axis to the right. After graphing, we'll find the intersection of the curves that we'll use as the limits of integration, and save the positive intersection as A. Since both functions are even, the graphs are symmetric and we can use +A and -A.





Now let's put the volume together one piece at a time.

The length of the base is $b = f(x) - g(x)$.

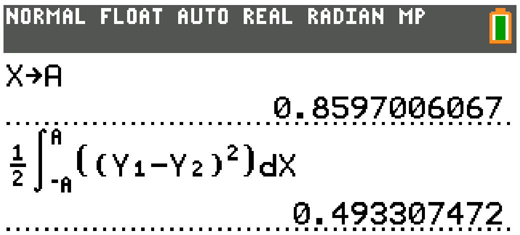
The area of each triangle is $A = \frac{1}{2}bh = \frac{1}{2}(f(x) - g(x))^2$.

The volume of the solid is

$$V = \int A\,dx$$

$$V = \frac{1}{2}\int_{-a}^{a} (f(x) - g(x))^2 \; dx$$

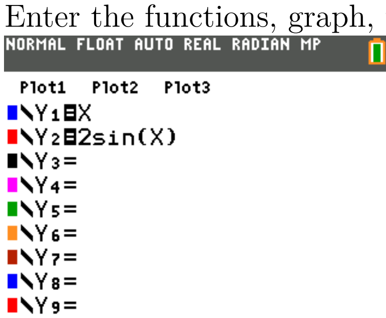
And entering that on the home screen gives:

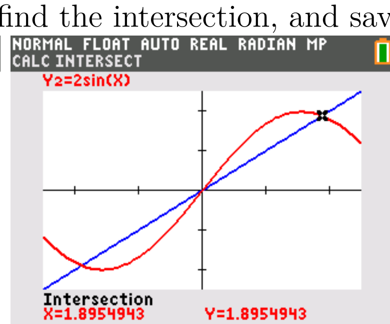


Example Find the volume of the region enclosed by $f(x) = x$ and $g(x) = 2\sin(x)$ revolved about the x-axis.

As always, graph first to visualize the problem. The region extends on both sides of the y-axis. When $x < 0$, $f(x) > g(x)$, but when $x > 0$, $g(x) > f(x)$, and clearly $f(0) = g(0)$. The easiest way to handle this is to note the symmetry, then to find the area on the right side of y, and multiply that by 2 for the final value.

Enter the functions, graph, find the intersection, and save as A.

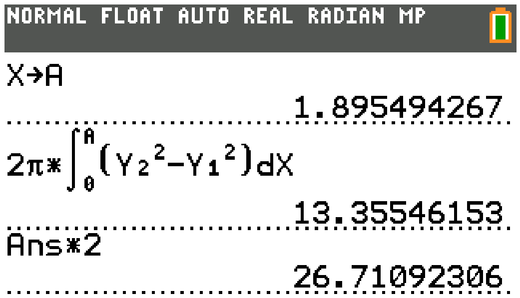




The volume using the disk / washer method (remember to square the inner and outer radii) is given by

$$V = 2\left(2\pi\int_{0}^{a} (g(x))^2 - (f(x))^2 \, dx\right)$$

Enter this directly using the function names Y_1 and Y_2.



4 Advanced

These examples are intended to capture some of the Calc BC specific skills.

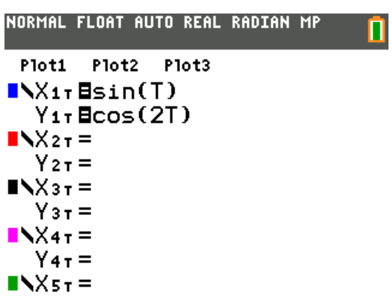
Example Find the length of the path on the curve defined by the parametric equations $x(t) = \sin t$ and $y = \cos 2t$ for $0 \leq t \leq 2\pi$.

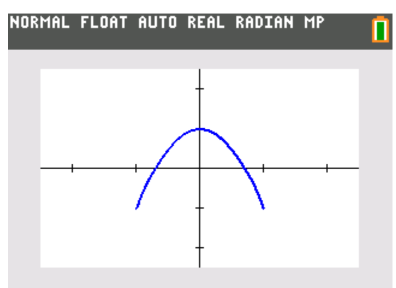
The arc length of a path on a parametric curve is given by

$$L = \int_{\alpha}^{\beta} \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$

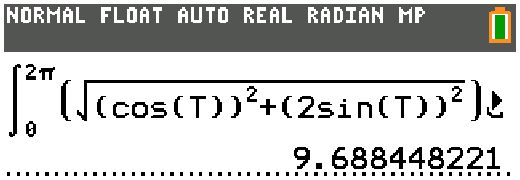
There's not much advantage to graphing the function first, but I like knowing how anyway! Set $\boxed{\text{mode}}$ to 'parametric', on the 'function' line. When in parametric mode the independent variable is automatically set to T.

Press $\boxed{\text{y=}}$ and enter the equations. Press $\boxed{\text{graph}}$. You can use $\boxed{\text{window}}$ to adjust the range of T displayed.



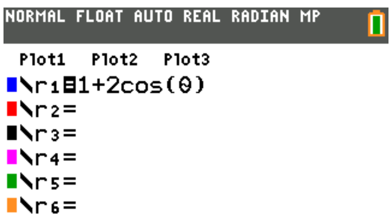


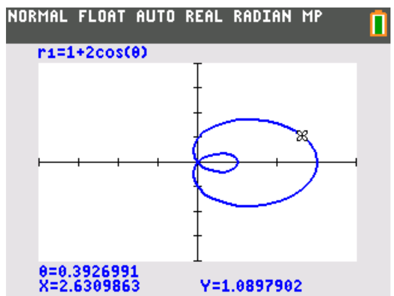
Take the derivatives symbolically (scratch paper) and enter the integral on the home screen.



Example Find the area of the smaller loop of $r = 1 + 2\cos(\theta)$.

Change the graphing mode to 'polar.' Enter the equation and graph.



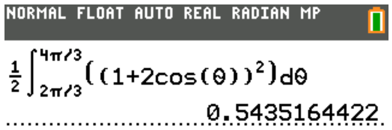


The $\boxed{\text{trace}}$ key and mode is especially useful! You can move the cursor around the curve and note the locations of zeros, maximums, intersections, etc. Here the first zero occurs at $\frac{2\pi}{3}$, the maximum of the little loop is at π and the second zero is at $\frac{4\pi}{3}$. The area of the inner loop is then given by

$$A = \int_a^b \frac{1}{2}(r(\theta))^2 \, d\theta$$

$$= \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1 + 2\cos\theta)^2 \, d\theta$$

This can be entered directly on the home screen.



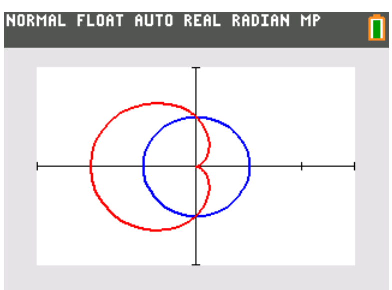
Example Find the area inside $r = 1$ and outside $1 - \cos(\theta)$.

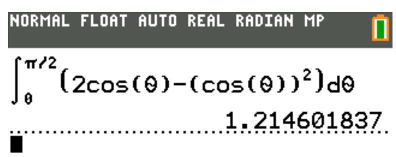
Graph the two curves to visualize. Since the graphs are both symmetric about the polar axis, we can simplify a bit. The area between the two curves is

$$A = \int_{-\pi/2}^{\pi/2} \frac{1}{2}(r_1^2 - r_2^2)\, d\theta$$

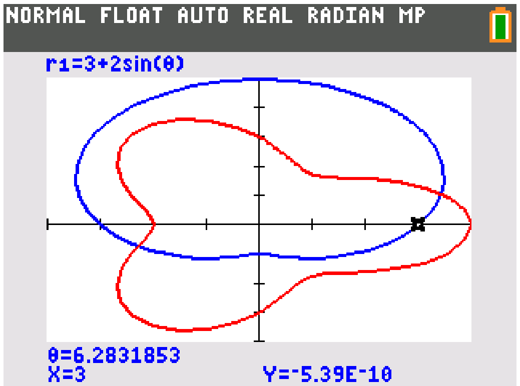
$$= 2\int_0^{\pi/2} \frac{1}{2}\left(1^2 - (1-\cos\theta)^2\right)\, d\theta$$

$$= \int_0^{\pi/2} \left(2\cos\theta - \cos^2\theta\right)\, d\theta$$



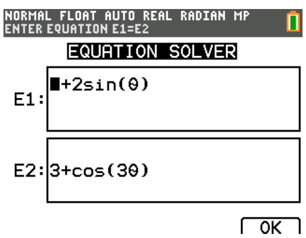


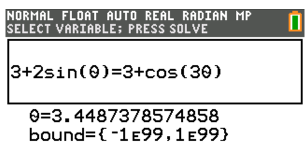
Example Consider the two polar curves given by $r = f(\theta) = 3 + 2\sin\theta$ and $r = g(\theta) = 3 + \cos(3\theta)$. Find the area enclosed by both curves.

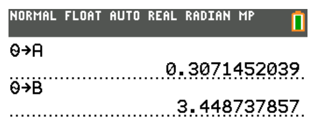


From the graph you can see we need to always choose the curve closest to the origin. We need to know the points of intersection of the curves, but since that feature is not supported by the graphing utility in polar mode, we'll use the solver.

Enter the two equations into the solver. Two different initial guesses are required to find the two intersections. Save the two intersections as A and B.

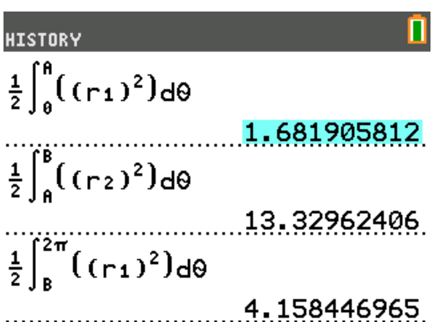


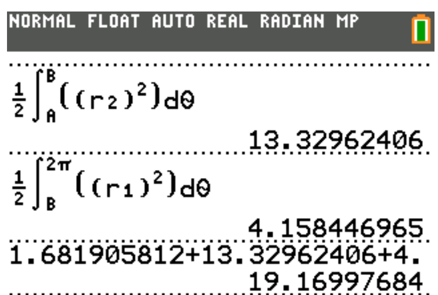




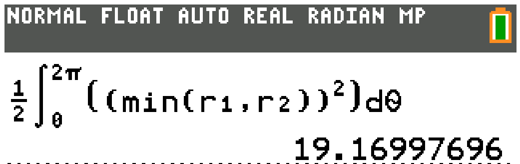
The total area can then be constructed as the sum of the three integrals:

$$A = \frac{1}{2}\int_0^A (f(\theta))^2\,d\theta + \frac{1}{2}\int_A^B (g(\theta))^2\,d\theta + \frac{1}{2}\int_B^{2\pi} (f(\theta))^2\,d\theta$$



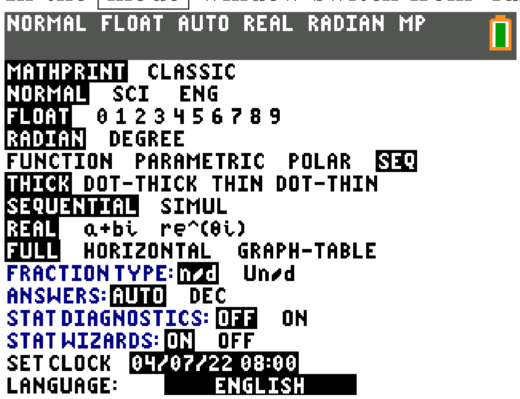


The calculator function, 'min()', selects the minimum of entries in a list. This seldom used function is a great shortcut for this problem.

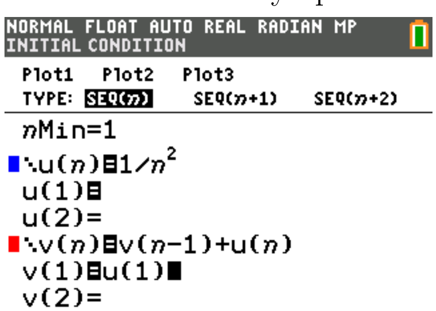


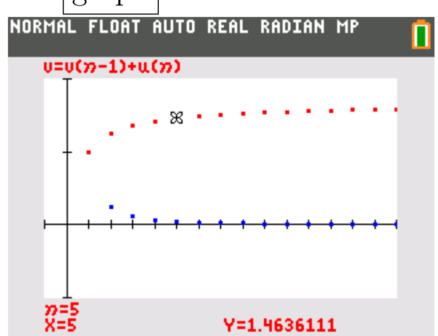
Example If you've made it this far, you're a fan of the power of calculators like me! Here is one last calculator function demonstrating graphing and evaluating sequences and series.

In the $\boxed{\text{mode}}$ window switch from 'function' or 'polar' to 'seq'.



For a sequence I'm using $1/n^2$ as an example, but any sequence can be entered into $u(n)$. To also show the partial sums, enter $v(1) = u(1)$ and $v(n) = v(n-1) + u(n)$ as shown. If you enter a new sequence into $u(n)$ the series will automatically update. Press $\boxed{\text{graph}}$.





Once $u(n)$ and $v(n)$ are entered, they can be evaluated directly from the home screen.