All work on this assessment should be your own. The technology allowed on this test includes: Desmos (`https://www.desmos.com/calculator`) and an approved TI calculator. This exam has 12 questions for a total of 60 points.

1. Let $f(x)$ be the function defined below:

$$f(x) = \begin{cases} \frac{x^2-2x+1}{x^2-6x+5} & \text{if } x \neq 1 \\ a & \text{if } x = 1 \end{cases}.$$

(a) (4 points) Find the value of a that makes $\lim_{x\to 1} f(x) = f(1)$.

(a) _____

(b) (2 points) If $a = 0$, what type of discontinuity would $f(x)$ have at $x = 1$?

(b) _____

2. (3 points) Below is the graph of $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ bounded between the graphs of x^2 and $-x^2$.

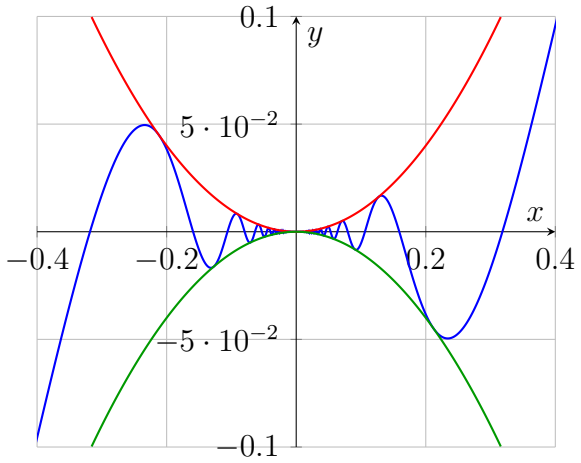

Figure 1: Graph of $x^2 \sin(\frac{1}{x})$ bounded between x^2 and $-x^2$

Complete the following sentence:

Figure 1 shows that $\lim_{x\to 0} \left(x^2 \sin\left(\frac{1}{x}\right)\right) =$ _____ *by the* _____ _____.

3. (2 points) Give the equation of a function that is everywhere continuous, but not differentiable at $x = 3$.

3. _____

4. (2 points) Given the equation of a function whose second derivative is constant.

4. _____

5. (2 points) Given the equation of a function that is infinitely differentiable.

5. _____

6. (5 points) Let $h(x) = (f \circ g)(x)$ and the following also be true about f, g, and, h:

$$h(a) = b, \qquad g(a) = c, \qquad f'(c) = d, \qquad g'(a) = \frac{1}{d}.$$

Calculate $h'(a)$.

6. _____

7. (7 points) On each sign chart, unlabeled tick marks are locations where $g'(x)$ or $g''(x)$ is zero and x-values where $g'(x)$ or $g''(x)$ does not exist are indicated by tick-marks labeled "DNE". Answer the proceeding questions regarding the characteristics of g using this information.

Sign Charts of g

(a) The graph of g is _____ on the interval $(-\infty, 2)$.

(b) The graph of g is _____ on the interval $(2, 4) \cup (4, \infty)$.

(c) The graph of g is concave _____ on the interval $(4, \infty)$.

(d) The graph of g is concave _____ on the interval $(-\infty, 4)$.

(e) The graph of g has a relative _____ on the interval $(-2, 3)$.

(f) The graph of g has a _____ point when $x = 2$.

(g) If $\lim_{x \to 4^-} g(x) = -\infty$ and $\lim_{x \to 4^+} g(x) = \infty$, then g has _____ discontinuity at $x = 4$.

8. (7 points) Thinking about L'Hôpital's Rule, determine which of the following forms are indeterminante.

◯ $\infty \cdot \infty$ ◯ $\frac{\infty}{\infty}$ ◯ 0^0 ◯ ∞^∞ ◯ $\frac{0}{0}$ ◯ 1^∞ ◯ $\infty - \infty$ ◯ $\frac{0}{\infty}$

◯ 0^∞ ◯ $\infty \cdot 0$ ◯ $\frac{\infty}{0}$ ◯ ∞^0

9. Below is the Riemann summation for a function $f(x)$ over some interval $[a, b]$.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{\sqrt{1-\left(\frac{\pi}{4}+\frac{\pi}{4n}k\right)^2}} \right) \cdot \frac{\pi}{4n}.$$

(a) (2 points) What is $f(x)$?

(a) _____

(b) (2 points) What is the interval over which $f(x)$ is being integrated?

(b) _____

(c) (3 points) Write the definite integral with the values of a and b and $f(x)$. DO NOT CALCULATE IT.

10. (9 points) Consider the two functions $f(x)$ and $g(x)$, which are graphed below.

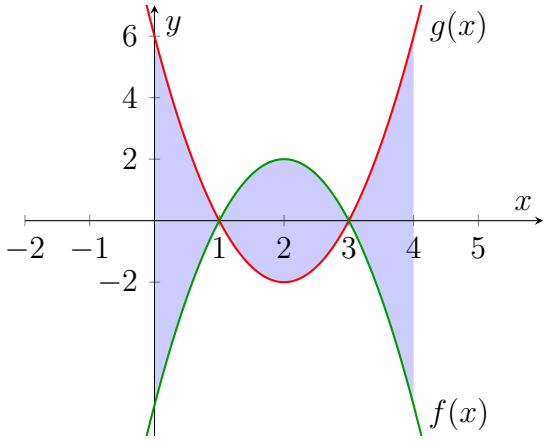

Figure 3: Graph of $f(x)$ and $g(x)$ with bounded area.

(a) Write a single definite integral using absolute values that would calculate the area bounded between f and g.

(b) Write THREE definite integrals without using absolute values that would calculate the area bounded between f and g.

11. (10 points) Let $h(x)$ be the piecewise defined function illustrated in the figure below. You should assume that $h(x)$ continues forever in the left and right direction. Moreover, let

$$H(x) = \int_0^x g(t)dt,$$

where the domain of $H(x)$ is $(-\infty, \infty)$.

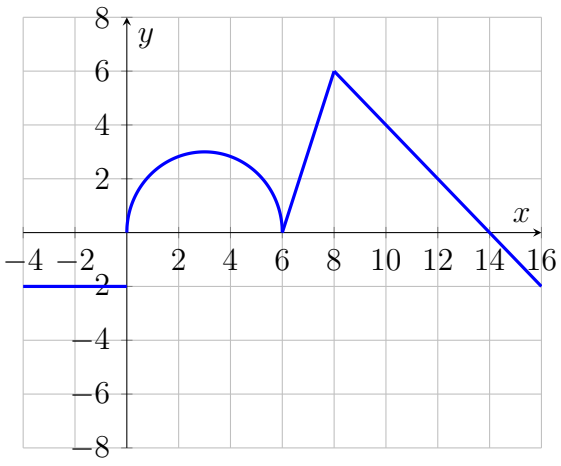

Figure 4: Graph of $h(x)$

(a) What are the critical numbers of $H(x)$?

(a) _____

(b) Over what interval(s) is $H(x)$ increasing?

(b) _____

(c) Over what interval(s) is $H(x)$ decreasing?

(c) _____

(d) For what value of x is $H(x)$ increasing the fastest?

(d) _____

(e) The function $H(x)$ has a local extrema at $x = 14$. What type of extrema is it?

(e) _____

12. Dear MAT 271 Student,

    It has been an honor and pleasure to teach you this semester. Of my classes, your Calculus I class has been the highlight of my semester. It is classes like yours that make me look forward to teaching. Furthermore, I hope that you have learned some amazing mathematics this semester and are ready to take on what ever path you choose from here. If you will be taking Calculus II here at CVCC in the Fall, I look forward to having you in my class! If not, I wish you all the best of luck in your future endeavours.

    So, here is how you get your extra credit! On the lines provided below (or if you did it before hand, on your own paper) tell a future MAT 271 student what they should do in order to be successful in MAT 271. Be as detailed as you can as I intend on sharing these with future students.

    Again, thank you for an awesome semester.

    Sincerely yours,

    E.A. Smith