Numerical Integration Assignment

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Numerical Integration Assignment

Question 0

Watch the lecture video here.

Did you watch the video? [Type yes or no.]

Question 1

The Golden Gate Bridge has a main span of 4,200 feet (the distance between the two towers). The main suspension cables that support the road over this span each form a parabolic shape. The length of each cable is found by

$$\text{Length} = \int_0^{21} 200\sqrt{1 + 0.000577x^2}\, dx$$

Part a

Approximate the value of this integral using left and right Riemann sums, the Midpoint Rule, the Trapezoidal Rule, and Simpson's Rule using $n = 10$, $n = 20$, and $n = 30$.

Part b

The actual value of this integral is approximately 4371.87. Observe how close each approximation comes to the right answer. In particular, notice how accurate Simpson's Rule proves to be, even though these values of n are relatively small.

Question 2

Consider the integral

$$\int_{-1}^{1.41} \frac{1}{x^2}\, dx$$

Part a

Use the Midpoint Rule with $n = 10$, $n = 50$, $n = 100$, and $n = 500$ to approximate this integral.

Part b

What happens to your approximations as n increases?

Notice that the function $\frac{1}{x^2}$ is unbounded at $x = 0$. This integral is divergent, so approximation won't work.

Question 3

Consider the function $f(x) = x^5 e^{-2x}$ over the interval $[-2, 2]$.

[Caution: Don't forget parentheses: e^(-2*x)]

Part a

Approximate the area under this curve using the Midpoint Rule, Trapezoidal Rule, and Simpson's Rule using $n = 100$, $n = 200$, and $n = 400$.

Part b

Rounding to one decimal place, this area is actually -363.2. Notice that Simpon's Rule is correct for all three values of n, but the Midpoint Rule is correct only for $n = 400$, and the Trapezoidal Rule is not correct for any of these values of n.

Question 4

Approximate $\int_{-1000}^{1000} \frac{1}{x^2 + 1}\, dx$ using Simpson's Rule. Hint: The interval width is 2,000, so pick an appropriate number of subintervals (n needs to be **big**).

What well-known number is this close to?